A SPATIAL INTERACTION MODEL FOR STATISTICAL ANALYSIS OF SOME INTERSPECIFIC COMPETITION EXPERIMENTS

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CERTIFICATE

Certified that the work entitled, ''A SPATIAL INTERACTION MODEL FOR STATISTICAL ANALYSIS OF SOME INTERSPECIFIC COMPETITION EXPERIMENTS'' has been carried out by Mr. M. Narayana Reddy under my supervision and has not been submitted elsewhere for the award of any other degree.

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Interspecific competition experiments involve growing two or more species together. When these species are grown in separate rows within a plot then it is called intercropping (IC). Among interspecific competition studies intercropping is one of the important topics (Mead, 1979). Currently there is a growing interest in IC studies because of its substantial advantages over monocropping (MC), i.e. growing them in separate plots. The importance and research needs of IC were reviewed by Willey (1979), while Mead and Riley (1981) reviewed the existing methods of statistical analysis. The conventional methods of statistical analysis associated with IC are not satisfactory due to the complex nature of interactions involved. Most of the methods suggested so far emphasize on testing the difference among the treatment means. However, for studying the effects of some important factors, such as spatial row arrangements and plant densities, fitting a relationship by incorporating these factors is important. This helps in arriving at the optimum row arrangement

and plant densities. In the present study efforts have been made for developing suitable relationships for two component crop experiments involving various row arrangements and plant densities of both the crops. In the first chapter motivation for the present study is discussed and the relevant literature is reviewed.

Different row arrangements in IC introduce different degrees of spatial interaction among the plants of the crops. It is of interest to find the row arrangement which maximizes a meaningful linear combination of two outputs. For this relationships are formulated for individual row yields based on the principles of plant competition. When r_1 rows of one crop is followed by r_2 rows of the other crop, for fixed intra and interrow distances, the following relationships are assumed :

$$y_{1i} = \eta_1 + \beta_{1k}\, y_{k,i-1} + \beta_{1m}\, y_{m,i+1} + u_{1i}, \quad i = 1,2,\ldots,r_1 ;$$

$k = 2$ when $i = 1$, $m = 2$ when $i = r_1$, otherwise $k = m = 1$ and $y_{ko} = y_{k,r_1+r_2}$,

$$y_{2j} = \eta_2 + \beta_{2k}\, y_{k,j-1} + \beta_{2m}\, y_{m,j+1} + u_{2j}, \quad j = r_1+1,\ldots,r_1+r_2 ;$$

$k = 1$ when $j = r_1+1$, $m = 1$ when $j = r_1+r_2$, otherwise $k = m = 2$ and $y_{m,r_1+r_2+1} = y_{m,1}$.

In the above equations η_k, $k = 1,2,$ is the expected row yield of crop k in the absence of competition ; y_{1i} and y_{2j} are the random variables corresponding to the yield of the first and second crops, respectively ; β_{kk} is the

competition coefficient between any two nearest neighbour (NN) rows of crop k ; β_{km} , $k \neq m = 1,2$ represents the competition of crop m on crop k ; u_{1i} and u_{2j} are errors and assumed to be independently distributed as $N(0,\sigma_1^2)$ and $N(0,\sigma_2^2)$, respectively.

The above equations belong to the simultaneous spatial process introduced by Whittle (1954). A bivariate model is obtained for the plot yields from the above set of equations. The parameters in this model are functions of row arrangement and proportion of the crops. Models for MC situations in terms of their plot yields are also developed. Some alternative formulations , such as conditional autoregressive model (Besag, 1974) , are also discussed. In the present situation simultaneous model appears to be more appropriate and hence it has been considered for further studies. The development of models for row arrangements is discussed in the second chapter.

The problem of estimation of parameters has been discussed in the third chapter. For this purpose the method of maximum likelihood (ML) is adopted in estimating the parameters. Estimators for replicated field experiments are obtained using the information from IC and MC plots. As the model is nonlinear iterative equations based on Newton-Raphson method for obtaining the ML estimates have been developed. Expressions for obtaining the large sample variance-covariance matrix are derived. The model has been extended to take care of block effects. To

examine the performance of the ML estimators , simulation study has been carried out by assuming some practical experimental situations. The effects of changing the error variances and the size of the experiment have also been studied.

In addition to row arrangements the magnitude of competition depends upon the level of plant densities. In the fourth chapter the model is extended to incorporate the parameters that quantify the effects due to plant densities. For this, the relationship between competition coefficient, say β , and the NN interplant distance x , is assumed as

$$\beta(x) = \beta_o x^{-c} ,$$

where β_o and c are constants. A method for obtaining the ML estimators of the parameters is developed. Expressions for large sample variance - covariance matrix are also derived . The results of a simulation study are also discussed.

In the fifth chapter some methods for obtaining the optimum row arrangement and plant densities are discussed . Certain linear combination of output such as land equivalent ratio (LER) , in terms of expected yields , are considered for optimization purposes . For obtaining the optimum row arrangement , comparison of expected output for various row arrangements , appears to be a convenient method . However , for obtaining optimum densities nonlinear optimization methods have to be used. These methods are illustrated with some examples.

In the sixth chapter the model has been fitted to an experimental data. Certain aspects of the adequacy of the fitted model to these data have been examined. The overall fit of the model appears to be satisfactory. The advantages of the fitted model , in gaining some insight in the nature of competition , and optimum row arrangement and plant densities , have been discussed.

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CHAPTER 1

INTRODUCTION REVIEW AND SCOPE

.1 INTRODUCTION

In plant ecology competition occurs among plants when they are onsuming limited resources. In such situations the presence of any lant changes the environment of its neighbouring plants which may ffect their growth rate and form. Different forms of competition xperiment and some of the related biometrical problems have been eviewed by Mead (1979). The purpose of competition studies is to nvestigate the competition relationships among plants of the same pecies or that of different species i.e. how the plants modify their ields due to joint utilization of the resources when they are grown ogether. These relationships add greatly in understanding crop rowth and production.

An interspecific competition study is the one in which plants f two or more species are grown together on the same plot. In such tudies usually yield of each species is recorded separately. The ost complex and important among the interspecific competition studies s intercropping. Growing two or more species together in separate ows in which various arrangements of rows are possible, is called ntercropping (IC). The importance of intercropping, its agronomic oncepts and research needed along with some concepts in the assessment f yield advantages, have been reviewed and discussed by Willey (1979). ne study herein is aimed at developing suitable statistical methodology or analysing the effects due to some of the important factors in ntercropping involving two species.

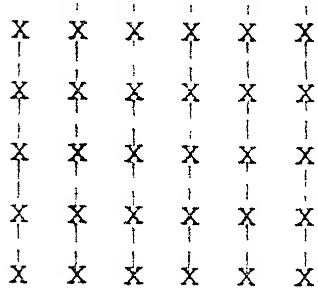
More generally, growing crops by irregular broadcasting or by mixing within the rows is called mixed cropping. Mixed cropping is an age-old cropping system in India and also in the semi-arid tropics throughout the world (Jodha,1979). The subsistance farmers in these areas practise mixed cropping because of its significant advantages. Some of them are the following :

1. It reduces the risk of total failure of crops per unit area in unfavourable weather conditions.
2. It utilizes the limited environmental resources like land, water and nutrients etc., more effeciently and may result in larger output than growing them separately on the same piece of land.

Intercroping is a modification of mixed cropping where each component crop is grown in separate rows.

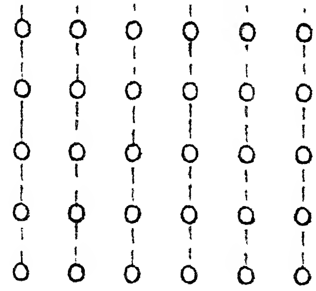
Component crop is either of the individual crops making up the IC ; intercrop yield is the yield of the component crop in IC expressed over the total intercropped area (i.e. area occupied by both the crops) ; Monocropping (MC) or sole cropping refers to a component crop being grown alone at optimum plant population and spacing , unless indicated otherwise. Some examples of IC and MC are shown in Figure 1.1.

The interrow (between rows) and intrarow (between plants within a row) distances for any component crop in IC need not be the same as that of MC. When an IC treatment is obtained by replacing certain proportion of a monocrop with an equivalent proportion of the other crop , it is termed as a replacement series treatment. Replacement series treatments



(d)

MC 1



(e)

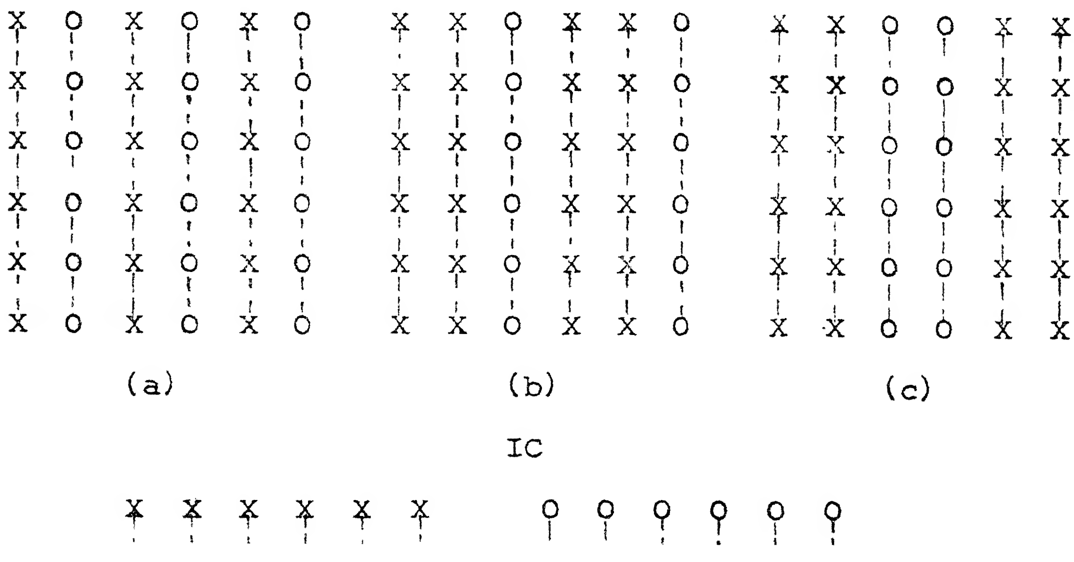
MC 2

X and O are the plants of two different species.

Fig. 1.1 Some examples of intercropping (IC) and monocropping (MC).

Normally traditional competition experiments include mixtures and monocrops of many species or genotypes, whereas IC experiment includes a large number of treatments for a given crop combination. In IC the common treatments are various spatial row arrangements (relative position of the rows) and plant densities of the component

are important in understanding and evaluating yield advantages of intercrops over monocrops. Figure 1.1 gives an example of a simple replacement series treatment where crop rows in a MC are replaced by the same number of rows of the other crop.



:rops. Two chief objectives of these experiments (Mead and Willey, 1980 'earce and Gilliver, 1978 ; Vandermeer, 1986) are the following :

.. To determine whether a given IC combination is indeed better than MC.

?. To understand the underlying mechanism in IC for enabling further improvement.

To meet these objectives a research worker is required to levelop appropriate models for analysing data from experiments .nvolving spatial row arrangements and plant densities. The present ;tudy concentrates on these aspects of the IC experiments.

.2 REVIEW OF THE LITERATURE

.2.1 Existing methods of statistical analysis

A good account on the current methods of statistical analysis 'f IC is discussed by Mead and Riley (1981) and these are further .llustrated by Dear and Mead (1983,1984). The statistical methods rhich are in current use are mostly for testing significant differences mong the treatments by analysis of variance (ANOVA) or covariance ANCOVA) techniques. Univariate methods have been mostly used for nalysing the individual crop yields and some derived variables such s monetary value and nutritive value. Other important variables rhich have been often analysed are some indices of the combined 'ields which characterise the competition. The most popular among .he competition indices of the combined yields is 'Land Equivalent :atio' (LER), as discussed by Mead and Willey (1980). LER is defined

as (Willey and Osiru,1972)

$$LER = L_1 + L_2 = Y_1/Y_{10} + Y_2/Y_{20}, \tag{1.2.1}$$

where Y_k, $(k = 1,2)$ is the yield of the component crop k in IC, Y_{ko} is its corresponding MC yield and L_k is the partial LER of crop k, i.e. proportion of the kth component crop yield to its monocrop yield. LER is widely used by agronomists because of its interpretation as ' the relative area required to produce the same yields that are achieved in IC through MC '. In the LER analysis one of the important hypotheses to be tested is H_o : LER = 1 against H_A : LER $\neq$ 1, and this is discussed by Skovgaard (1986). When H_o is accepted the yield advantages in IC are not significantly different from MC.

In IC the growth rate of any component crop is not independent of the other. Hence analysis of IC is essentially a bivariate problem, as analysis by univariate methods may not detect the real treatment differences. Pearce and Gilliver (1978) suggested a standard multivariate technique of analysis of dispersion for testing the treatment effects. They have demonstrated the use of graphical procedures for comparing treatment means for the two species simultaneously. These graphical procedures have been extended (Pearce and Gilliver,1979; Gilliver and Pearce,1983)(1) to compare the advantages of IC over MC, (2) to identify the IC system that maximizes the total produce when there are certain constraints on the component crop yields, and (3) to study the two and three factor interactions in factorial experiments. The analysis suggested by Pearce and Gilliver is based on the assumption that the correlation

coefficient remains the same for all the treatments which may not be true in many situations. To overcome this Singh and Gilliver (1988) suggested an extension to the analysis suggested by Pearce and Gilliver by considering the correlated error structure among the treatments. When the IC treatments include certain factors such as various spatial row arrangements and plant densities of the component crops, it is important to develop appropriate response relations among the component crops yield rather than looking at the significance of certain treatmental differences. Such relationships will be of great help in understanding the response of yield in IC to varying spatial row arrangements and plant densities. This relationship may be useful in finding the optimum IC treatment.

Vandermeer (1986) suggested a relationship in terms of plant yields of component crops to predict the yields for different spatial arrangements of the plants in IC. The suggested equation is

$$\omega_{1i} = \delta_{1i} - \sum_{\substack{j=1 \\ j \neq i}}^{N_1} \beta'_{1ij}\omega_{1j} - \sum_{j=1}^{N_2} \beta'_{2ij}\omega_{2j} , \qquad (1.2.2)$$

where ω_{1i} is the yield of the ith plant of the first crop ; δ_{1i} is the expected yield attainable in the absence of competition ; β'_{1ij} represents the competition effect of the jth plant of the first crop on the ith plant of the same crop ; β'_{2ij} represents the competition effect of the jth plant of the second crop on the ith plant of the first crop, and ω_{2j} is the yield of the jth plant of the second crop. Some adhoc methods have been used to estimate the

parameters in the above equation through the individual plant data from small systematic designs. In IC the total yield of a plot (or the average plant yield) is important, hence the statistical analysis based on the model that explains the variation among plot yields of the component crops, is more realistic.

Regarding the yield density relationship Wright (1981) suggested the following models for analysing the experimental data of crop mixtures,

$$Y_1 = d_1(\alpha_1 + \beta_{11}d_1 + \beta_{12}d_2)^{-1} + \varepsilon_1 \tag{1.2.3a}$$

$$Y_2 = d_2(\alpha_2 + \beta_{22}d_2 + \beta_{21}d_1)^{-1} + \varepsilon_2 \tag{1.2.3b}$$

where Y_k is the yield per unit area of crop k, d_1 and d_2 are the plant densities of crop 1 and crop 2, respectively ; $\beta_{k,m}$ (k,m = 1,2) are the competition parameters, ε_1 and ε_2 are the errors. Here Y_1 and Y_2 are independent which may not be true in IC.

Another important aspect of IC analysis is to compare the stability of IC treatments on crop yields and monetary value over years. This aspect is not considered in the present study. Some of the papers dealing with this analysis are by Pearce and Edmondson (1982,1984), Mead etal. (1986) and Singh etal. (1988). The designs that are currently in use are randomized block design (RBD), split plot and systematic designs (Mead and Stern, 1980).

As the present study is concerned with the statistical modelling and analysis of IC experimental data, the methods for analysing

raditional competition experiments like crop mixtures are not eviewed here. Some of the papers in this area are by Williams (1962), cGilchrist (1965), McGilchrist and Trenbath (1971), Federer (1979) nd Federer etal. (1976).

.2.2 Spatial interaction models

.2.2.1 Introduction

When a set of spatially located variables X_i , $i = 1,2,\ldots,n$; lisplay interdependence over space then the data are spatially utocorrelated. The autocorrelation may be due to reaction or interaction among the variables. Adjacent trees may compete for resources like water, nutrients and sunlight etc., displaying between tree interactive effects, but they may also react to the general availability of nutrients within the reach of their root system. When reaction is dominant, a regression model is appropriate, whereas interactive effects suggest the need for a model with a spatially dependent covariance structure. The model should include the parameters of reactive and interactive effects as both are important factors. As the present study is concerned with developing the statistical models for IC , based on spatial interaction among the plants, the spatial interaction models are briefly reviewed in the following section.

1.2.2.2 Simultaneous model

Whittle (1954) proposed a simultaneous spatial autoregressive scheme of the following form

$$X_i = \mu_i + \sum_{j\neq i} s_{ij}(X_j - \mu_j) + \varepsilon_i \qquad i,j = 1,2,\ldots,n\,; \qquad (1.2.4)$$

where $E(\varepsilon_i) = 0$ and $\mathrm{cov}(\varepsilon_i,\varepsilon_j) = \sigma_i^2$ if $i = j$, zero otherwise.

the matrix notation this can be expressed as

$$\underline{X} = \underline{\mu} + S(\underline{X} - \underline{\mu}) + \underline{\varepsilon} , \qquad (1.2.5)$$

ere $\underline{X} = (X_1, \ldots, X_n)'$, $\underline{\mu} = (\mu_1, \ldots, \mu_n)'$

$\underline{\varepsilon} = (\varepsilon_1, \ldots, \varepsilon_n)'$, $S = (s_{ij})_{nxn}$ and $s_{ii} = 0$.

e then have

$$E(\underline{\varepsilon}) = 0, \; V(\underline{\varepsilon}) = \Lambda = \text{diag } (\sigma_1^2 , \ldots, \sigma_n^2).$$

$$E(\underline{X}) = \underline{\mu} , \; V_S(\underline{X}) = (I-S)^{-1} \Lambda (I-S')^{-1}. \qquad (1.2.6)$$

he necessary and sufficient condition for the existence of his process is $(I-S)$ should be nonsingular. In the present study t is assumed that $\underline{X}$ is distributed as multivariate normal with ean $\underline{\mu}$ and variance V_S.

.2.2.3 Conditional model

This approach requires to specify the conditional distribution f each random variable X_i given $X_j = x_j$, $j \neq i$. In the case f nearest neighbour situation this approach was first suggested y Bartlett (1955) and the corresponding mathematical and statistical theory was developed by Besag (1974,1975). Suppose that the conditional distribution of X_i is normal with conditional mean

$$E(X_i | X_j = x_j , j \neq i) = \mu_i + \sum_{j \neq i} c_{ij} (x_j - \mu_j) \qquad (1.2.7)$$

and conditional variance

$$\text{Var } (X_i \mid X_j = x_j , j \neq i) = \sigma_i^2 ,$$

where $c_{ii} = 0$, then the dispersion matrix of the scheme is given by

$$V_C = (I-C)^{-1} \Lambda \text{ , where } C = (c_{ij})_{n \times n} \text{ .} \qquad (1.2.8)$$

The scheme exists only where V_C is symmetric and positive definite. Besag (1974) named the above model as auto-normal model. In fact he introduced a general class of auto-models based on the conditional probability formulation. Here $\underline{X}$ has a multivariate normal distribution with mean $\underline{\mu}$ and variance V_C.

2.2.2.4 Difference between simultaneous and conditional models

The general distinction between these two approaches is due to Brook (1964). The difference lies in the specification of the dispersion matrix. In the case of Gaussian process Besag (1974) discussed a simple method of deriving an equivalent conditional scheme from simultaneous scheme. Bartlett (1975) discussed the same through spectral density functions. In the case of Gaussian process when $\Lambda = \sigma^2 I$, the simultaneous and conditional schemes will be identical only if (Riply,1981)

$$(I - C) = (I - S')(I - S).$$

In the case of simultaneous model there is no unique representation of dispersion matrix in terms of S, whereas in the conditional case the dispersion matrix determines C uniquely . But the major disadvantage with conditional approach is about their existence because of the condition that $\Lambda^{-1}(I-C)$ should be symmetric and positive definite. These two models are named as spatial autoregressive models analoguous to those in time series.

1.2.2.5 Moving average models

As in the case of time series (Box and Jenkins, 1970) moving average (MA) models can be specified by taking $E(X_i) = 0$ without any loss of generality, as follows (Cliff and Ord, 1981),

$$X_i = \varepsilon_i + \sum_{j \neq i} a_{ij}\varepsilon_j \ , \quad i,j = 1,2,\ldots,n \ ; \tag{1.2.9}$$

where $E(\varepsilon_i) = 0$, $Var(\varepsilon_i) = \sigma_i^2$ and $Cov(\varepsilon_i, \varepsilon_j) = 0$ where $i \neq j$. In the matrix form these can be expressed as

$$\underline{X} = (I+A)\underline{\varepsilon} \ , \ \text{where } A = (a_{ij})_{n \times n} \ ,$$

$$Var(\underline{X}) = (I+A)\ \Lambda\ (I+A'). \tag{1.2.10}$$

The process is representable by simultaneous spatial process only if the largest eigen value of A is less than unity in absolute value. Haining (1978a) has discussed in detail about these schemes. MA models have been mentioned often in literature but rarely used in practice (Haining, 1978b) in spatial studies.

1.2.3 Some applications of spatial interaction models and methods of estimating parameters

1.2.3.1 Applications in field experiments

Spatially autocorrelated data occur in a wide variety of scientific disciplines, especially in ecology and geography. A good description on analysing these type of data is given in the books by Bartlett (1975), Riply (1981), Cliff and Ord (1981) and Upton and Fingleton (1985). Recently there has been a considerable interest in the applications of spatial autoregressive models in agricultural field trials. Whittle (1954) and Besag (1974,1977a)

fitted these models to analyse uniformity trial data. This type of analysis helps agronomists in planning their future experiments. Making use of autoregressive models Bartlett (1978) reexamined theoretically the method suggested by Papadakis (1937), for adjusting the treatment means based on the neighbouring plot residuals.

Mead (1966,1967,1968,1971) adopted simultaneous approach to study the plant competitions in MC through small competition experiments. He fitted the following model for the data from systematic design with hexagonal arrangement of plants (Mead,1967) :

$$\omega_i = \delta + \beta' \sum_{j(i)} (\omega_j - \delta) + e_i \,, \qquad (1.2.11)$$

where j(i) is the set of neighbours of the ith plant, ω_i is the yield of the ith plant, δ is the population mean, β is the competition coefficient among the neighbouring plants and e_i's are errors which are independently distributed with zero mean and variance σ^2.

Kempton (1982) fitted simultaneous model for incorporating the competition effects due to neighbouring plots in the variety trials. The model is

$$Y_{ir} = \tau_r + \beta \sum_{j(i)} Y_j/p + \varepsilon_{ir} \,, \qquad (1.2.12)$$

where j(i) is the set of p neighbours of plot i, Y_{ir} is the yield of the rth variety in the ith plot, β is the common competition coefficient among the neighbouring plots, and errors ε_{ir}'s are

independently and normally distributed with zero mean and variance σ^2. The above model with block parameters to account for large scale fertility differences has been fitted by Besag and Kempton (1986)

1.2.3.2 Estimation procedures

In the case of simultaneous models ordinary least squares estimators are inconsistent as pointed by Whittle (1954). Ord (1975) suggested a modified least squares method which yields consistent but inefficient estimators, whereas in conditional models least squares estimators are consistent. Besag (1972,1974,1975) suggested coding and pseudo-likelihood (Besag, 1975, 1977b) methods for conditional models. In the normal cases pseudo-likelihood estimators correspond to the least squares estimators. In the case of maximum likelihood method, the log-likelihood for simultaneous and conditional models represented by the equations (1.2.5) and (1.2.7), when $\sigma_i^2 = \sigma^2$ $(i = 1,2,\ldots,n)$ is given by

$$2\log L = -n \log (2\sigma^2) + \log|B| - \sigma^{-2}(\underline{X}-\underline{\mu})' B(\underline{X}-\underline{\mu}), \qquad (1.2.13)$$

where $B = (I-S')(I-S)$ for simultaneous model and $B = (I-C)$ for conditional model, and $|B|$ is the determinant of B.

If $\underline{\mu}$ is of the form $D\underline{\theta}$, where D is the design matrix, the maximum likelihood estimators of $\underline{\theta}$, σ^2 and the parameters in B, are given by

$$\hat{\underline{\theta}} = (D'\hat{B}D)^{-1} D'\hat{B}\,\underline{X} \qquad (1.2.14)$$

$$\hat{\sigma}^2 = n^{-1}(\underline{X}-D\hat{\underline{\theta}})' \hat{B}(\underline{X}-D\hat{\underline{\theta}}). \qquad (1.2.15)$$

The estimates of parameters in B may be obtained by minimizing

$$-n^{-1} \log |\hat{B}| + \log \{ n^{-1}(\underline{X}-D\hat{\underline{\theta}})'\hat{B}(\underline{X}-D\hat{\underline{\theta}}) \}. \qquad (1.2.16)$$

Estimation of $\underline{\theta}$ and B proceeds iteratively by successive use of equation (1.2.14) and minimization of (1.2.16). Computationally the problem arises in the evaluation of $|B|$ at each stage when minimizing (1.2.16) iteratively. When B involves only one parameter e.g. $B = \beta W$, where W is the matrix with known coefficients, Ord (1975) evaluated $|B|$ as,

$$|B| = \prod_{i=1}^{n} (1-\beta\lambda_i) ,$$

where λ_i's are the eigen values of W. Kempton (1982) also used this method in evaluating $|B|$. Mead (1967) in small sample competition studies evaluated $|B|$ by Lanczo's (1957) method of interpolation which is based on Chebyshev polynomials. In the case of simple simultaneous models involving one or two parameters Whittle (1954) used general Fourier inversion method for evaluating $|B|$ when large number of observations are available on lattice systems.

1.3 SCOPE OF THE PRESENT STUDY

When the resources are uniformly distributed then the yield variations of the component crops in IC are mainly due to the changes in the degree of plant interactions among the plants of the crops and random errors. Hence the statistical analysis based on spatial interaction models which incorporate the spatial dependence into their covariance structure, reviewed in section 1.2.2, appears to be more suitable for analysing IC experimental data.

In the second chapter model incorporating effects due to various row arrangements of the component crops, based on the principles of plant competition is developed. Model is formulated in terms of individual row yields. Model for plot yields is derived from the model formulated in terms of row yields since plot yields are usually observed in IC experiments. Model corresponding to MC is also derived. In section 2.3 some alternative models are formulated. Similtaneous model appears to be more suitable for this situation. Hence this model is considered in details in the subsequent chapters.

Estimation of the parameters through a replicated field experiment is discussed in the third chapter for the simultaneous model. The method of Maximum likelihood (ML) is used for estimating the parameters. The ML estimators are derived in section 3.2 using information from IC and MC plots. As the model is nonlinear iterating equations based on Newton-Raphson method have been developed in this section. Expressions for asymptotic variance-covariances of the estimators are derived in section 3.3. The model is extended to incorporate the block effects in section 3.4 and corresponding estimators are derived. The performance of ML estimators are examined through simulation study and the results of this study are discussed in section 3.5. The effects of change in error variances and size of the experiment are also studied in this section.

In the fourth chapter the model is extended to incorporate the parameters that quantify the effects due to plant densities.

This extension is discussed in section 4.2. Model corresponding to MC is also developed. ML estimators for this extended model are derived in section 4.3. In section 4.4 block effects are incorporated into the model. The results of the simulation study carried out for examining the performance of the ML estimators are discussed in section 4.5. A particular case of the model for variation only due to plant densities is discussed in section 4.6. A numerical method for obtaining the ML estimates by iteration and the expressions for the asymptotic variance-covariances of the estimators are given in the appendix of this chapter.

In the fifth chapter some methods for obtaining the optimum row arrangement and plant density levels, using the models developed in earlier chapters, are discussed. These aspects are discussed in section 5.2. Some examples are given illustrating the method.

In the sixth chapter the model developed in the fourth chapter has been fitted to some experimental data. Some aspects of the goodness of fit of the model are also examined. The interpretation of the fitted competition coefficients and their use in giving some indication regarding the optimum row arrangement and density levels are also discussed.

1.4 NOTATION AND ABBREVIATIONS

ML	maximum likelihood
IC	intercropping
MC	monocropping
NN	nearest neighbour
LER	land equivalent ratio
LER_e	land equivalent ratio in expected yields
MER	monetary equivalent ratio
MER_e	monetary equivalent ratio in expected yields
L_k	partial land equivalent ratio of kth crop
L_{ke}	partial land equivalent ratio of kth crop in expected yields
TMV	total monetary value

CHAPTER II

SPATIAL INTERACTION MODELS FOR CROPS WITH DIFFERENT ROW ARRANGEMENTS

2.1 INTRODUCTION

The advantage of IC over MC is due to the adjustability of the component crops in a given environment such that the total competition pressure per unit area is reduced. This suggests building up a model for IC systems in terms of parameters which quantify the competition effects in different situations. The reduction in total competition pressure in IC is due to less intensity of interspecific competition than intraspecific competition (Trenbath, 1974; Willey, 1979). In other words the intensity of plant interactions between species is less than that of within species. The arrangement of the plants on the plot is a basic factor that modifies total competition pressure. Hence spatial arrangement of the crop rows is one of the major factors that is being studied in IC research programme currently. Some appropriate spatial interaction models in terms of the competition parameters for the row arrangement of the component crops in IC have been developed and discussed in this chapter.

2.2 DEVELOPMENT OF SIMULTANEOUS MODEL

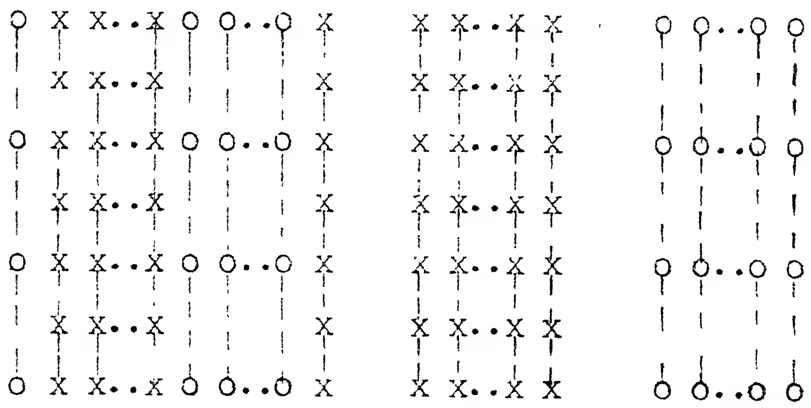
2.2.1 First-order simultaneous autoregressive model

Let us consider a plot with N_1 rows of crop 1 and N_2 rows of crop 2. The arrangement of these $N = (N_1+N_2)$ rows are such that r_1 rows of crop 1 are followed by r_2 rows of crop 2

Fig. 2.1 Arrangement of r_1+r_2 rows in intercropping (IC) and monocropping (MC).

For simplicity we assume that interrow distances are same in IC and MC. We shall consider the replacement series treatment where intrarow distances may differ for two component crops but are same as in their corresponding MC. This situation is illustrated in Figure 2.1 for r_1+r_2 crop rows. The IC treatment is obtained by replacing r_2 rows of species 1 in MC treatment by r_2 rows of species 2 or vice versa. As the respective inter and intrarow distances in IC are equal to those in MC the yield of any row depends on the type of neighbouring rows. This subsequently depends upon the spatial row arrangement of component crops in the plot. The competition

Moreover, $b(r_1+r_2) = N_1+N_2 = N$, $r_1 = 1,2,\ldots,N_1$, $r_2 = 1,2,\ldots,N_2$, and b is an integer such that above condition is satisfied.



r_1 r_2 r_1+r_2 r_1+r_2

IC MC 1 MC 2

for resources is likely to be severe between nearest neighbouring (NN) rows. In developing the model the following assumptions are made.

1. The competition is limited only to the NN rows and its effect is linear on the NN row yields.
2. The first and the last rows are neighbours.

To implement the last condition we assume that appropriate guard rows are taken in field experiment.

Following these assumptions the individual row yields in an $r_1 : r_2$ arrangement of the component crops can be expressed in terms of the first-order simultaneous stochastic equations as

$$
\begin{aligned}
y_{11} &= \eta_1 + \beta_{11} y_{12} + \beta_{12} y_{2,r_1+r_2} + \varepsilon_{11} \\
y_{12} &= \eta_1 + \beta_{11} y_{13} + \beta_{11} y_{11} + \varepsilon_{12} \\
\vdots \quad & \quad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\
y_{1,r_1-1} &= \eta_1 + \beta_{11} y_{1,r_1-2} + \beta_{11} y_{1r_1} + \varepsilon_{1,r_1-1} \\
y_{1,r_1} &= \eta_1 + \beta_{11} y_{1,r_1-1} + \beta_{12} y_{2,r_1+1} + \varepsilon_{1r_1} \qquad (2.2.1) \\
y_{2,r_1+1} &= \eta_2 + \beta_{21} y_{1r_1} + \beta_{22} y_{2,r_1+2} + \varepsilon_{2,r_1+1} \\
y_{2,r_1+2} &= \eta_2 + \beta_{22} y_{2,r_1+1} + \beta_{22} y_{2,r_1+3} + \varepsilon_{2,r_1+2} \\
\vdots \quad & \quad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\
y_{2,r_1+r_2-1} &= \eta_2 + \beta_{22} y_{2,r_1+r_2-2} + \beta_{22} y_{2,r_1+r_2} + \varepsilon_{2,r_1+r_2-1} \\
y_{2,r_1+r_2} &= \eta_2 + \beta_{22} y_{2,r_1+r_2-1} + \beta_{21} y_{11} + \varepsilon_{2,r_1+r_2} .
\end{aligned}
$$

These equations can be written as

$$y_{1i} = \eta_1 + \beta_{1k}y_{k,i-1} + \beta_{1m}y_{m,i+1} + u_{1i} , \quad i = 1,2,\ldots,r_1 ; \qquad (2.2.2a)$$

$k = 2$ where $i = 1$, $m = 2$ when $i = r_1$, otherwise $k=m=1$

and $y_{k,o} = y_{k,r_1+r_2}$;

$$y_{2j} = \eta_2 + \beta_{2k}y_{k,j-1} + \beta_{2m}y_{m,j+1} + u_{2j}, \quad j = r_1+1,\ldots,r_1+r_2 ; \qquad (2.2.2b)$$

$k = 1$ when $j = r_1+1$, $m = 1$ when $j = r_1+r_2$, otherwise $k=m=2$ and $y_{m,r+r_2+1} = y_{m,1}$.

In the above equations η_k is the expected row yield of crop k in the absence of competition ; y_{1i} and y_{2j} are the random variables corresponding to the yields of ith and jth rows of first and second crop , respectively ; β_{kk} is the competition coefficient between any two NN rows of crop k ; β_{km} , $k \neq m = 1,2$ represents the competition of crop m on crop k ; u_{1i} and u_{2j} are random errors and assumed to be independently distributed as $N(0,\sigma_1^2)$ and $N(0,\sigma_2^2)$, respectively.

For convenience equations (2.2.1) can be expressed in matrix form as

$$(I-B)\underline{y} = \underline{\eta} + \underline{u} , \qquad (2.2.3)$$

where $\underline{\eta}$, $\underline{y}$ and $\underline{u}$ are (r_1+r_2) component column vectors and B is (r_1+r_2) square matrix given by

$\underline{\eta} = (\eta_1,\eta_1,\ldots,\eta_1,\eta_2,\ldots,\eta_2)'$, $\underline{y} = (y_{11},y_{12},\ldots,y_{1r_1},y_{2,r_1+1},\ldots y_{2,r_1+r_2}$

rather than of individual row. In the following we shall consider a model in terms of the total yield of r_1 rows of crop 1 and r_2 rows of crop 2 which can be derived by summing the individual component crop rows yields. This is easily done by premultiplying (2.2.3) by A', where

$$\underset{(2\times(r_1+r_2))}{A'} = \begin{bmatrix} 1 & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 1 & 1 & \cdots & 1 \end{bmatrix},$$

giving

$$A'(I-B)\underline{y} = A'\underline{\eta} + A'\underline{u} \,. \qquad (2.2.4)$$

This can be expressed as

$$A'\underline{y} = A'\underline{\eta} + A'B\underline{y} + A'\underline{u}$$

$$\underline{u} = (u_{11}, u_{12}, \ldots, u_{1r_1}, u_{2,r_1+1}, \ldots, u_{2,r_1+r_2})'$$

$$B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

$$\underset{(r_k x r_k)}{B_{kk}} = \begin{bmatrix} 0 & \beta_{kk} & 0 & \cdots & 0 & 0 \\ \beta_{kk} & 0 & \beta_{kk} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & \beta_{kk} \\ 0 & 0 & 0 & \cdots & \beta_{kk} & 0 \end{bmatrix} ; \underset{(r_k x r_m)}{B_{km}} = \begin{bmatrix} 0 & 0 & 0 & \cdots & \beta_{km} \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 0 \\ \beta_{km} & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$k \neq m$; $k, m = 1, 2$.

In IC the objective is to maximize the yield of a plot

$$Y_{11} + Y_{1r_1} = 2Y_1/r_1 , \qquad (2.2.6a)$$

and

$$Y_{2,r_1+1} + Y_{2,r_1+r_2} = 2Y_2/r_2 , \qquad (2.2.6b)$$

equations (2.2.5a,b) reduce to

$$Y_1 = r_1\eta_1 + 2(r_1-1)r_1^{-1}\beta_{11}Y_1 + 2r_2^{-1}\beta_{12}Y_2 + \varepsilon_1 ,$$

$$Y_2 = r_2\eta_2 + 2r_1^{-1}\beta_{21}Y_1 + 2(r_2-1)r_2^{-1}\beta_{22}Y_2 + \varepsilon_2 ,$$

which can be expressed as

$$(1-2(r_1-1)r_1^{-1}\beta_{11})Y_1 = r_1\eta_1 + 2r_2^{-1}\beta_{12}Y_2 + \varepsilon_1 \qquad (2.2.7a)$$

$$(1-2(r_2-1)r_2^{-1}\beta_{22})Y_2 = r_2\eta_2 + 2r_1^{-1}\beta_{21}Y_1 + \varepsilon_2 . \qquad (2.2.7b)$$

i.e.

$$Y_1 = r_1\eta_1 + \beta_{11}(y_{11} + 2y_{12} + \cdots + 2y_{1,r_1-1} + y_{1r_1}) + \beta_{12}(y_{2,r_1+1} + y_{2,r_1+r_2}) + \varepsilon_1 \tag{2.2.5a}$$

$$Y_2 = r_2\eta_2 + \beta_{21}(y_{11}+y_{1r_1}) + \beta_{22}(y_{2,r_1+1}+2y_{2,r_1+2}+\cdots+2y_{2,r_1+r_2-1}+y_{2,r_1+r_2}) + \varepsilon_2 \tag{2.2.5b}$$

Here

$$Y_k = \sum_{i=1}^{r_k} y_{ki} \quad ; \quad \varepsilon_k = \sum_{i=1}^{r_k} u_{ki} .$$

By using the approximation

These equations may be written as

$$(I-H)\underline{Y} = P\underline{\mu} + \underline{\varepsilon} , \qquad (2.2.8)$$

where $\underline{Y} = (Y_1, Y_2)'$, $\underline{\mu} = (\mu_1, \mu_2)'$, $\underline{\varepsilon} = (\varepsilon_1, \varepsilon_2)'$, $\mu_k = (r_1+r_2)\eta_k$ and

$$H = 2\begin{bmatrix} (r_1-1)r_1^{-1}\beta_{11} & r_2^{-1}\beta_{12} \\ r_1^{-1}\beta_{21} & (r_2-1)r_2^{-1}\beta_{22} \end{bmatrix} ; \quad P = \begin{bmatrix} p_1 & 0 \\ 0 & p_2 \end{bmatrix} ,$$

$p_k = r_k/(r_1+r_2)$, $k = 1, 2$.

For r_1 and $r_2 \leq 2$ equations (2.2.7a,b) are exact and there is no need of approximations (2.2.6a,b). In practical situations r_1 and r_2 are likely to be small.

Alternatively , the H matrix can be easily obtained directly from the parameters that quantify the total competition, i.e.

$$A'B\underline{1} = \begin{bmatrix} 2(r_1-1)\beta_{11} + 2\beta_{12} \\ 2\beta_{21} + 2(r_2-1)\beta_{22} \end{bmatrix} = H'A\underline{1} , \qquad (2.2.9)$$

where $\underset{(1\times(r_1+r_2))}{\underline{1}'} = (1,1,\ldots,1)$.

The equations for average yield per row are given by

$$(1-2(r_1-1)r_1^{-1}\beta_{11})\bar{Y}_1 = \eta_1+2\beta_{12}r_1^{-1}\beta_{12}\bar{Y}_2+\bar{\varepsilon}_1 \qquad (2.2.10a)$$

$$(1-2(r_2-1)r_2^{-1}\beta_{22})\bar{Y}_2 = \eta_2+2\beta_{21}r_2^{-1}\beta_{21}\bar{Y}_1+\bar{\varepsilon}_2 , \qquad (2.2.10b)$$

where $\bar{Y}_k = Y_k/r_k$ and $\bar{\varepsilon}_k = \varepsilon_k/r_k$.

The total number of NN rows for r_1 rows of first crop are $2r_1$, among which $2(r_1-1)$ rows are of the first crop and two are of the second crop.

In the case of MC the yield of ith row of crop k can be represented by

$$y_{ki} = \eta_k + \beta_{11}(y_{k,i-1} + y_{k,i+1}) + u_{1i},\ i = 1,2,\ldots,r_1+r_2 \ . \tag{2.2.11}$$

The corresponding equation for the total yield is

$$Y_{ko} = \mu_k + 2\beta_{kk}Y_{ko} + \varepsilon_{ko} ,$$

where $Y_{ko} = \sum_{i=1}^{r_1+r_2} y_{ki}$, $\varepsilon_{ko} = \sum_{i=1}^{r_1+r_2} \varepsilon_{ki}$.

The above equation can be written as

$$(1 - 2\beta_{kk})Y_{ko} = \mu_k + \varepsilon_{ko}. \tag{2.2.12}$$

The extension to the plot with b repetitions of (r_1+r_2) rows is straightforward. The equation given by (2.2.8) holds good with the following changes ;

$$Y_k = \sum_{i=1}^{N_k} y_{ki} , \quad \varepsilon_k = \sum_{i=1}^{N_k} u_{ki} \text{ and } \mu_k = N_k\eta_k \ .$$

From equation (2.2.8)

$$E(\underline{Y}) = (I-H)^{-1} P\underline{\mu} , \quad V(\underline{Y}) = (I-H)^{-1} P^{1/2} VP^{1/2} (I-H')^{-1} \tag{2.2.13}$$

where $P^{1/2} = \begin{bmatrix} \sqrt{p_1} & 0 \\ 0 & \sqrt{p_2} \end{bmatrix}$, $V = \begin{bmatrix} V_1 & 0 \\ 0 & V_2 \end{bmatrix}$, and $V_k = (r_1+r_2)\sigma_k^2$.

For the validity of equation (2.2.8), (I-H) should be nonsingular. In the case of MC

$$E(Y_{ko}) = \mu_k(1-2\beta_{kk})^{-1} \text{ and } V(Y_{ko}) = V_k(1-2\beta_{kk})^{-2} .$$

For example, in 2:2 row arrangement , i.e. $r_1 = r_2 = 2$, the first - order simultaneous stochastic equations , as described by (2.2.1) , are as the following :

$$y_{11} = \eta_1 + \beta_{11}y_{12} + \beta_{12}y_{22} + u_{11}$$

$$y_{12} = \eta_1 + \beta_{11}y_{11} + \beta_{12}y_{21} + u_{12}$$

$$y_{21} = \eta_2 + \beta_{21}y_{12} + \beta_{22}y_{22} + u_{21}$$

$$y_{22} = \eta_2 + \beta_{22}y_{21} + \beta_{21}y_{11} + u_{22}$$

i.e.

$$(I-B)\underline{y} = \underline{\eta} + \underline{u} ,$$

where $y = (y_{11}, y_{12}, y_{21}, y_{22})'$; $\eta = (\eta_1, \eta_1, \eta_2, \eta_2)'$;

$u = (u_{11}, u_{12}, u_{21}, u_{22})'$

and

$$B = \begin{bmatrix} 0 & \beta_{111} & 0 & \beta_{12} \\ \beta_{11} & 0 & \beta_{12} & 0 \\ 0 & \beta_{21} & 0 & \beta_{22} \\ \beta_{21} & 0 & \beta_{22} & 0 \end{bmatrix} .$$

$$\text{Here } A'B = \begin{bmatrix} \beta_{11} & \beta_{11} & \beta_{12} & \beta_{12} \\ \beta_{21} & \beta_{21} & \beta_{22} & \beta_{22} \end{bmatrix} = \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} = HA',$$

$$\text{where } H = \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix}, \quad A' = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}.$$

The equation for the total yield is

$$(I-H)\underline{Y} = P\,\underline{\mu} + \varepsilon ,$$

where $Y_k = \sum_{i=1}^{2} Y_{ki}$, $p_k = \mu_k/2, \mu_k = 4\eta_k$, $\varepsilon_k = \sum_{i=1}^{2} u_{ki}$.

The model (2.2.8) also holds for certain situations where the interrow distances in MC 1 and MC 2 are different . For example , let x_{11} and x_{22} be the interrow distances in MC 1 and MC 2 , respectively , such that $b_1x_{11} = b_2x_{22}$, where b_1 and b_2 are integers. The area occupied by b_1 rows of crop 1 is equal to the area occupied by b_2 rows of crop 2. Let us consider an IC system where every set of b_1 rows of crop 1 is replaced by a set of b_2 rows of crop 2 in MC 1 and vice versa . In IC the interrow distance between the crop 1 and crop 2 is $(x_{11}+x_{22})/2$. We only consider the arrangements r_1, r_2 for which $b_1b_2^{-1}r_2$ is an integer. In this case the model (2.2.8) holds with p_1 and p_2 as the following :

$$p_1 = r_1/(r_1 + b_1b_2^{-1}r_2) \; ; \quad p_2 = r_2/(b_2b_1^{-1}r_1 + r_2) \; . \quad (2.2.14)$$

2.2.2 Second-order simultaneous autoregressive model

Consider the situation in which yield of a row depends linearly on the yields of first and second NN rows. In $r_1 : r_2$ arrangement the total number of neighbours for $r_1 + r_2$ rows of crop 1 and crop 2 are $4(r_1 + r_2)$. Componentwise, the first and the second-order neighbours of the same and different crops are as following :

	Crop 1		Crop 2		
	First-order	second-order	First-order	second-order	Total
Crop 1	$2(r_1-1)$	$2(r_1-2)$	2	4	$4r_1$
Crop 2	2	4	$2(r_2-1)$	$2(r_2-2)$	$4r_2$

The matrix $A'B\,\underline{1}$ can now be expressed as

$$A'B\,\underline{1} = HA'\,\underline{1} = \begin{bmatrix} 2(r_1-1)\beta_{11} + 2(r_1-2)\beta_{11}^{(2)} + 2\beta_{12} + 4\beta_{12}^{(2)} \\ 2\beta_{21} + 4\beta_{21}^{(2)} + 2(r_2-1)\beta_{11} + 2(r_2-2)\beta_{22}^{(2)} \end{bmatrix},$$

where

$$H = \begin{bmatrix} \{2(r_1-1)\beta_{11} + 2(r_1-2)\beta_{11}^{(2)}\}r_1^{-1} & \{2\beta_{12}+4\beta_{12}^{(2)}\}r_2^{-1} \\ \{2\beta_{21} + 4\beta_{21}^{(2)}\}r_1^{-1} & \{2(r_1-1)\beta_{22}+2(r_2-2)\beta_{22}^{(2)}\}r_2^{-1} \end{bmatrix}$$

$\beta_{km}^{(2)}$ is the competition coefficient between y_{ki} and $y_{m,i+2}$ $k,m = 1,2$. In the terms of total yield the equation is same as (2.2.8) except for H which is replaced by the one given

ve. In some cases it may be realistic to assume some ationship between first and second-order competition parameters h as

$$\beta_{11}^{(2)} = \beta_{11}/a_1 \, , \; \beta_{12}^{(2)} = \beta_{12}/a_2 \, , \; \beta_{22}^{(2)} = \beta_{22}/a_3 \text{ and } \beta_{21}^{(2)} = \beta_{21}/a_4 \, ,$$

re a_1, a_2, a_3 and a_4 are known constants.

n the total number of competition parameters to be estimated aains same as in the case of the first-order model. For example 2:2 row arrangement

$$B = \begin{bmatrix} 0 & \beta_{11} & 2\beta_{12}^{(2)} & \beta_{12} \\ \beta_{11} & 0 & \beta_{12} & 2\beta_{12}^{(2)} \\ 2\beta_{21}^{(2)} & \beta_{22} & 0 & \beta_{21} \\ \beta_{21} & 2\beta_{21}^{(2)} & \beta_{22} & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} \beta_{11} & \beta_{11} & 2\beta_{12}^{(2)}+\beta_{12} & 2\beta_{12}^{(2)}+\beta_{12} \\ 2\beta_{21}^{(2)}+\beta_{21} & 2\beta_{21}^{(2)}+\beta_{21} & \beta_{22} & \beta_{22} \end{bmatrix} = \begin{bmatrix} \beta_{11} & 2\beta_{12}^{(2)}+\beta_{12} \\ 2\beta_{21}^{(2)}+\beta_{21} & \beta_{22} \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$= HA' .$$

3 SOME ALTERNATIVE MODELS

3.1 Conditional model

Following the notation used in section 1.2.2, let the nditional distribution of Y_i be normal with conditional mean

$$E(Y_i \mid \text{all others}) = \mu_i + \sum_{j \neq i}^{n} c_{ij} Y_j \, , \qquad (2.3.1)$$

The joint distribution of $\underline{Y}$ is multivariate normal (Besag, 1974) with

$$E(\underline{Y}) = \underline{\theta} = (I-C)^{-1}\,\underline{\mu} \text{ and } V(\underline{Y}) = (I-C)^{-1}\Lambda . \qquad (2.3.2)$$

The yields of the component crop rows y_{1i} and y_{2j}, $i = 1, 2, \ldots, r_1$; $j = r_1+1, \ldots, r_1+r_2$, can be expressed in terms of the first-order conditional model as,

$$E(y_{1i} \mid \text{all others}) = \eta_1 + \beta_{1k}\, y_{k,i-1} + \beta_{1m}\, y_{m,i+1} \quad ;(2.3.3a)$$

$k = 2$ where $i = 1$; $m = 2$ when $i = r_1$; otherwise $k = m = 1$

and

$$E(y_{2i} \mid \text{all others}) = \eta_2 + \beta_{2k}\, y_{k,j-1} + \beta_{2m}\, y_{m,j+1} \quad ;(2.3.3b)$$

$k = 1$ when $j = r_1+1$; $m = 1$ when $j = r_1+r_2$; otherwise $k = m = 2$.

and conditional variance

$$V(Y_i \mid \text{all others}) = \sigma_i^2 .$$

The conditional expected value can be written as

$$E(Y_i \mid \text{all others}) = \mu_i + \sum_{j \neq i} c_{ij}\theta_j + \sum_{j \neq i} c_{ij}(y_j - \theta_j)$$

$$= \theta_i + \sum_{j \neq i} c_{ij}(y_j - \theta_j),$$

where

$$\theta_i = \mu_i + \sum_{j \neq i} c_{ij}\,\theta_j \quad ,$$

i.e. $\quad \underline{\theta} = (I-C)^{-1}\,\underline{\mu}\,, \quad \underline{\theta} = (\theta_1, \ldots, \theta_n)'.$

The vector of (r_1+r_2) row yields $\underline{y}$ follows a multivariate normal distribution with

$$E(\underline{y}) = (I-B)^{-1}\,\underline{\eta} \quad \text{and} \quad V(\underline{y}) = (I-B)^{-1}\,\Sigma \;, \qquad (2.3.4)$$

where $\underset{(r_1+r_2)x(r_1+r_2)}{\Sigma} = \text{diag}\,(\Sigma_1, \Sigma_2)$, $\underset{(r_k x r_k)}{\Sigma_k} = \text{diag}\,(\sigma_k^2)$.

Using the methods similar to that of simultaneous model the distribution of total yields of r_1 rows of crop 1 and r_2 rows of crop 2 is a bivariate normal with

$$E(\underline{Y}) = (I-H)^{-1}\,P\underline{\mu} \quad \text{and} \quad V(\underline{Y}) = (I-H)^{-1}\,PV \;. \qquad (2.3.5)$$

The necessary and sufficient condition for the validity of the above model is $(I-H)^{-1}\,PV$ should be symmetric and positive definite. The symmetric property of the matrix $V^{-1}P^{-1}(I-H)$ leads to the condition

$$\beta_{12}\,\sigma_1^2 = \beta_{21}\,\sigma_2^2 \;. \qquad (2.3.6)$$

This condition restricts the use of the conditional model considerably.

2.3.2 Moving average model

The moving average formulation (1.2.10) in terms of the individual row yields , is

$$\underline{y} = \underline{\eta} + A\underline{u} + \underline{u} \;; \qquad (2.3.7)$$

where $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$,

$$A_{kk} = \begin{bmatrix} 0 & \alpha_{kk} & 0 & \cdots & 0 & 0 \\ \alpha_{kk} & 0 & \alpha_{kk} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \alpha_{kk} & \cdots & 0 & \alpha_{kk} \\ 0 & 0 & 0 & \cdots & \alpha_{kk} & 0 \end{bmatrix} \qquad A_{km} = \begin{bmatrix} 0 & 0 & \cdots & 0 & \alpha_{km} \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \\ \alpha_{km} & 0 & \cdots & 0 & 0 \end{bmatrix}$$
$$(r_k \times r_k) \qquad\qquad\qquad\qquad (r_1 \times r_2)$$

In terms of the total yields the model is

$$\underline{Y} = P\underline{\mu} + (I+F)\ \underline{u}\ , \qquad (2.3.8)$$

where

$$F = \begin{bmatrix} 2(r_1-1)r_1^{-1}\alpha_{11} & 2r_2^{-1}\alpha_{12} \\ 2r_1^{-1}\alpha_{21} & 2(r_2-1)r_2^{-1}\alpha_{22} \end{bmatrix}.$$

Under the same assumptions about u's as in simultaneous model $\underline{Y}$ follows a bivariate normal distribution with

$$E(\underline{Y}) = P\underline{\mu} \text{ and } V(\underline{Y}) = (I+F)\ P^{1/2}\ V\ P^{1/2}\ (I+F'). \qquad (2.3.9)$$

In experiments where competition plays a dominant role it is more realistic to consider the dependence of y_i on the phenotypic values of the neighbouring rows rather than on residuals. Numerical iterative method is more complicated for estimation of the parameters in MA model in comparison to simultaneous model since inversion of (I+F) is involved in each stage of iteration.

2.3.3 Regression model

One can also think of using the regression model by assuming that the yield of any row depends linearly on the true but unknown expected yield (η_k) of the neighbouring rows in the absence of competition. Following the same notation used earlier the row yields in $(r_1 : r_2)$ arrangement for the first-order dependence can

where $p_1, p_2, \mu_1, \mu_2, \varepsilon_1$ and ε_2 are the same as defined earlier.

The above equations can be written as

$$Y_1 = p_1\mu_1' + 2(r_1+r_2)^{-1}\beta_1 + \varepsilon_1 \qquad (2.3.11a)$$

$$Y_2 = p_2\mu_2' + 2(r_1+r_2)^{-1}\beta_2 + \varepsilon_2 , \qquad (2.3.11b)$$

where $\mu_1' = \mu_1(1+2\beta_{11})$, $\mu_2' = \mu_2(1+2\beta_{22})$,

$$\beta_1 = \beta_{12}\mu_2 - \beta_{11}\mu_1 \text{ and } \beta_2 = \beta_{21}\mu_1 - \beta_{22}\mu_2 .$$

Let us consider a linear combination of Y_1 and Y_2 as

$$Y = a_1Y_1+a_2Y_2 = \alpha_1p_1+\alpha_2p_2+2(r_1+r_2)^{-1}\beta + \varepsilon , \qquad (2.3.12)$$

where $\alpha_1 = a_1\mu_1'$, $\alpha_2 = a_2\mu_2'$, $\beta = a_1\beta_1 + a_2\beta_2$ and $\varepsilon = a_1\varepsilon_1 + a_2\varepsilon_2$.

be expressed as

$$\underline{y} = \underline{\eta} + B\underline{\eta} + \underline{u} , \qquad (2.3.10)$$

where $\underline{y}$, $\underline{\eta}$ and $\underline{u}$ are the column vectors of observable row yields, expected row yields in the absence of competition, and errors , respectively , for the two crops. B is a matrix with competition coefficients having the same structure as in the simultaneous model (section 2.2).

The equations in terms of total yields Y_1 and Y_2 are

$$Y_1 = p_1\mu_1 + 2(r_1-1)r_1^{-1}\beta_{11}p_1\mu_1 + 2r_2^{-1}\beta_{12}p_2\mu_2 + \varepsilon_1 \qquad (2.3.10a)$$

$$Y_2 = p_2\mu_2 + 2r_1^{-1}\beta_{21}p_1\mu_1 + 2(r_2-1)r_2^{-1}\beta_{22}p_2\mu_2 + \varepsilon_2, \qquad (2.3.10b)$$

The equation (2.3.12) is similar to the linear mixture model (Scheffe',1958) with additional factor $2(r_1+r_2)^{-1}$. This additional factor represents the degree of intimacy of the two species. For example, it is unity if the species are perfectly intimate, and this happens when $r_1 = r_2 = 1$. The coefficients α_1 and α_2 represent the effects of proportions and β represents the effect of the row arrangement. Nelder (1963) suggested the use of mixture models for analysing the competition experiments such as random mixtures involving grass species.

From equations (2.3.10a,b) the mean and variances of $\underline{Y}$ are

$$E(\underline{Y}) = (I-H)^{-1}P\underline{\mu} \text{ and } V(\underline{Y}) = P^{1/2}\, VP^{1/2} \quad . \tag{2.3.13}$$

Here the expected value of $\underline{Y}$ is same as in the case of simultaneous and conditional models, but the variance differs and is free from competition parameters. Estimation of β coefficients by least squares in this case involves minimizing the quantity,

$$(\underline{Y}-(I-H)^{-1}\, P\underline{\mu})'\; P^{1/2}\, VP^{1/2}(\underline{Y}-(I-H)^{-1}\, P\underline{\mu}). \tag{2.3.14}$$

2.4 DISCUSSION

Let us consider a MC experimental plot with N plants, and suppose the yield, ω_i, of the ith plant can be represented as

$$\omega_i = \delta + \beta' \sum_{j\neq i} W_{ij}\, \omega_j + e_i \;,\; i = 1,2,\ldots, N \;; \tag{2.4.1}$$

where δ is the expected plant yield in the absence of competition, β' is the common competition coefficient and e_i's are errors with usual assumptions. W_{ij} is the weight assigned to the jth plant in relation to the ith plant.

which may depend on the distance d_{ij} between them. For simplicity we take $W_{ij} = 1$ if $d_{ij} \leq d$, zero otherwise, where d is some fixed distance. Let N plants be distributed over the plot in such a way that there are N_1 plants within the neighbourhood d. By summing the individual plant yields, the yield per plot can be expressed as

$$Y = N\delta + (N_1 - 1)\beta' Y + \varepsilon , \qquad (2.4.2)$$

where $$Y = \sum_{i=1}^{N} y_i , \ \varepsilon = \sum_{i=1}^{N} e_i .$$

Since the neighbourhood area A is constant then for sufficiently large N, $N_1 - 1 \simeq A\rho$, where ρ is the plant density, i.e. the number of plants per unit area. Hence from (2.4.2) we have

$$E(Y) = N\delta/(1-\beta\rho) , \qquad (2.4.3)$$

where $\beta = \beta' A$. In terms of expected plant yield

$$E(\omega) = \delta/(1-\beta\rho) = 1/(a+b\rho) \qquad (2.4.4)$$

where $a = \delta^{-1}$, $b = -\beta\delta^{-1}$.

This is a well established asymptotic plant density relationship (Bleasdale and Nelder, 1960 ; Shinozaki and Kira, 1956). Vandermeer (1984a, 1984b) discussed a similar type of competition theory as above for developing the relation (2.4.4).

The competition model (2.4.1) described above is the same as Mead's (1967) model (1.2.11). Hence from this competition model we have obtained well established plant density relationship.

However, there is a basic difference, i.e. in competition model individual plant yields are to be considered for estimating the parameters whereas in plant density studies average plant yields or total plot yields are used. Moreover, there is a difference in the error structures which has implications in the estimation of the parameters. Proceeding with the other models such as conditional and regression, one does not obtain the classical plant density relationship given by (2.4.4). Thus, the use of simultaneous model has an advantage over the others. The simultaneous models (2.2.1) suggested in this study is a natural extension of the competition model (2.4.1) in MC. The procedure used to obtain the model (2.2.8) for total yield for IC is same as the procedure used to obtain the classical plant density relation (2.4.4). The model (1.2.2) suggested by Vandermeer (1986) in terms of plant yields is similar to (2.2.1) with higher order competition coefficients but it lacks appropriate stochastic formulation. Kempton (1982) and Besag and Kempton (1986) also used the simultaneous model (1.2.12) to incorporate the interplot competition in varietal trials.

The restriction $\beta_{12}\sigma_1^2 = \beta_{21}\sigma_2^2$ in the case of conditional model is a serious drawback in IC situations. In general β_{12} and β_{21} will be of different degree and there is no reason to assume $\beta_{12}/\beta_{21} = \sigma_2^2/\sigma_1^2$. The conditional model may be appropriate

in MC situation where there is only one competition coefficient involved. There is hardly any difference between simultaneous and conditional formulations when competition coefficient is small , as has been pointed out by Besag and Kempton (1986). In IC situations , however, both the models are different.

MA model in spatial process is an alternative way of representing the simultaneous model with reparameterisation of the coefficient matrix. However, it is difficult to interpret the parameters in the MA model whereas in the simultaneous model they have a natural interpretation. We have obtained the regression model (2.3.11) by making the yield of any row depending upon η , the expected rows yield in the absence of competition. We have seen that this is same as linear mixture model (Scheffé, 1958) with an additional parameter representing the degree of intimacy between the species. As suggested by Nelder (1963) this type of model may be appropriate in analysing the experimental data from random mixtures involving grass species.

In the IC experiments the competition plays a dominating role. Thus, it is more realistic to think that yield of any row depends upon the actual yield of the neighbouring rows rather than on their expected values, as in the case of regression models. Hence simultaneous model appears to be more appropriate in IC situation. For using the model (2.2.8) it is necessary to estimate the parameters through an experiment of reasonable

size. These aspects are discussed in the next chapter. In the proposed model the yield of any row depends linearly on the neighbouring row yields. In case of grain yield this assumption may not be entirely valid. In such situations it may be more realistic to consider yield of any row depending linearly on other characteristics such as vegetative growth of the neighbouring rows. However, the model considered here may not be unrealistic in case yields of the neighbouring rows are proportional to the vegetative growth. Normally β coefficients will be negative due to competition for resources. However, there may be situations where the interspecific competition coefficients β_{12} or β_{21} may be positive, i.e. the presence of another crop is beneficial to the first crop. This has been more appropriately termed as cooperation.

The model (2.2.8) can be used in arriving at the strategies of the row arrangements for achieving the maximum output per unit area. This is discussed in the fifth chapter in more detail. In practice, however, there may be other factors such as plant density and different fertilizer treatments which are of considerable importance. The simultaneous model suggested in this chapter for analysing the IC experimental data involving row arrangements can be extended without difficulty to these situations. For example, the model can be extended to study the effect of plant density on crop yields by choosing β coefficients as appropriate functions of intra and interrows distances. Some of these aspects are discussed in the fourth chapter. Introducing different genotypes

of the crops or more number of crops will increase the number of competition parameters. These aspects are not included in the present study.

CHAPTER III

FITTING OF SIMULTANEOUS MODEL

3.1 INTRODUCTION

In this chapter the details of estimation of the parameters in the first-order simultaneous model when the data are collected from a replicated field experiment , is discussed. Since the least squares estimators are inconsistent for simultaneous models, as pointed by Whittle (1954) and further discussed by Ord (1975) , maximum likelihood method (ML) is adopted in estimating the parameters. This method is discussed in section 3.2. Since the estimating equations are nonlinear in parameters iterative procedure for obtaining the estimates is developed. The expressions for large sample variance-covariances of the estimators are derived in section 3.3. For eliminating the systematic fertility differences block parameters are introduced and the estimating equations are derived in section 3.4 . In section 3.5 the performance of the estimators are examined by some simulation studies.

3.2 ESTIMATION OF THE PARAMETERS

3.2.1 Maximum likelihood estimators

Consider a two component IC experiment with t row arrangements and two MC plots arranged in a completely randomized design . Let there be N rows in each plot with b_i repetitions of $r_{1i} : r_{2i}$ rows of component crops $(i = 1,2,\ldots,t)$. Each treatment is

replicated K times. In section 2.2 it has been shown that the plot yields of the component crops in IC form the following bivariate sets of simultaneous stochastic equations :

$$(1 - 2(r_{1i}-1) r_{1i}^{-1} \beta_{11}) Y_{1ij} = p_{1i}\mu_1 + 2r_{2i}^{-1}\beta_{12}Y_{2ij} + \varepsilon_{1ij} \tag{3.2.1a}$$

$$(1 - 2(r_{2i}-1) r_{2i}^{-1} \beta_{22}) Y_{2ij} = p_{2i}\mu_2 + 2r_{1i}^{-1}\beta_{21}Y_{1ij} + \varepsilon_{2ij} \tag{3.2.1b}$$

$$i = 1, 2, \ldots, t \; ; \; j = 1, 2, \ldots, K \; ;$$

where $\varepsilon_{kij} = \sum_{m=1}^{N_k} u_{kijm}$, $\mu_k = N\eta_k$, $N_{1i} + N_{2i} = N$, $N_{ki} = b_i \, r_{ki}$, Y_{kij} and ε_{kij} are the yield and error associated with the plot in the jth replication of the ith treatment of crop k ; u_{kijm} is the error associated with the mth row in the jth replication of the ith row arrangement of crop k. Based on the assumptions about the errors, u's , as mentioned in section 2.2 , $\underline{Y}_{ij} = (Y_{1ij}, Y_{2ij})'$ follows a bivariate normal distribution with mean and variance as

$$E(\underline{Y}_{ij}) = (I-H_i)^{-1} P_i\underline{\mu} \; ; \; V(\underline{Y}_{ij}) = (I-H_i)^{-1} P_i^{1/2} V P_i^{1/2} (I-H_i')^{-1} \tag{3.2.2}$$

where

$$H_i = 2 \begin{bmatrix} (r_{1i}-1) r_{1i}^{-1} \beta_{11} & r_{2i}^{-1} \beta_{12} \\ r_{1i}^{-1} \beta_{21} & (r_{2i}-1) r_{2i}^{-1} \beta_{22} \end{bmatrix} ,$$

$P_i^{1/2} = \text{diag}\,(\sqrt{p_{1i}}, \sqrt{p_{2i}})$, $V = \text{diag}\,(V_1, V_2)$, $V_k = N\sigma_k^2$, $p_{ki} = r_{ki}/(r_{1i}+r_{2i})$, $k = 1, 2$.

In the case of MC

$$E(Y_{koj}) = \mu_k(1-2\beta_{kk})^{-1}, \quad V(Y_{koj}) = V_k(1-2\beta_{kk})^{-2}, \qquad (3.2.3)$$

where Y_{koj} is the yield of the plot from the jth replication in MC of kth crop.

$$-2\log L = \text{constant} - 2K[\sum_i \log |G_i| + \log \{(1-2\beta_{11})(1-2\beta_{22})\}]$$
$$+ K(t+1)\log(V_1 \cdot V_2) + \sum_i \sum_j (G_i Y_{ij} - \underline{\mu})' V_i^{-1}(G_i \underline{Y}_{ij} - \underline{\mu})$$
$$+ \sum_k [V_k^{-1} \sum_j \{(1-2\beta_{kk})Y_{koj} - \mu_k\}^2], \qquad (3.2.4)$$

where $G_i = P_i^{-1}(I-H_i)$, $V_i = V P_i^{-1}$.

By assuming β_{11}, β_{12}, β_{22} and β_{21} known, the ML estimates of μ_1, μ_2, V_1 and V_2 are obtained by equating partial derivatives of -2log L to zero, and they are

$$K(1+\sum_i p_{1i})\hat{\mu}_1 = \sum_i \sum_j \{(1-2(r_{1i}-1)r_{1i}^{-1}\beta_{11})Y_{1ij} - 2r_{2i}^{-1}\beta_{12}Y_{2ij}\}$$
$$+ (1-2\beta_{11})\sum_j Y_{1oj}, \qquad (3.2.5a)$$

$$K(1+\sum_i p_{2i})\hat{\mu}_2 = \sum_i \sum_j \{(1-2(r_{2i}-1)r_{2i}^{-1}\beta_{22})Y_{2ij} - 2r_{1i}^{-1}\beta_{21}Y_{1ij}\}$$
$$+ (1-2\beta_{22})\sum_j Y_{2oj}, \qquad (3.2.5b)$$

$$K(1+t)\hat{V}_1 = \sum_i \sum_j p_{1i}^{-1}\{(1-2(r_{1i}-1)r_{1i}^{-1}\beta_{11})Y_{1ij} - 2r_{2i}^{-1}\beta_{12}Y_{2ij} - p_{1i}\hat{\mu}_1\}^2$$
$$+ \sum_j \{(1-2\beta_{11})Y_{1oj} - \hat{\mu}_1\}^2 \qquad (3.2.6a)$$

$$K(1+t)\hat{V}_2 = \sum_i \sum_j p_{2i}^{-1}\{(1-2(r_{2i}-1)r_{2i}^{-1}\beta_{22})Y_{2ij} - 2r_{1i}^{-1}\beta_{21}Y_{1ij} - p_{2i}\hat{\mu}_2\}^2$$
$$+ \sum_j \{(1-2\beta_{22})Y_{2oj} - \hat{\mu}_2\}^2. \qquad (3.2.6b)$$

By substituting these values (3.2.4) reduces, in terms of competition coefficients, as

$$f(\underline{\beta}) = \text{constant} - 2K \left[\sum_i \log |G_i| + \log\{(1-2\beta_1)(1-2\beta_{22})\} \right] + K(t+1) \log (\hat{V}_1 \cdot \hat{V}_2), \qquad (3.2.7)$$

where $\underline{\beta}' = (\beta_{11}, \beta_{12}, \beta_{22}, \beta_{21})$.

The ML estimate of $\underline{\beta}$ is the value corresponding to the minimum value of the function (3.2.7).

3.2.2 A Numerical Method for finding ML Estimates

From equation (3.2.7) the ML estimate of $\underline{\beta}$ is the value corresponding to the minimum value of the function

$$f_1(\underline{\beta}) = \log \prod_k \{\sum_i \sum_j p_{ki}^{-1} e_{kij}^2 + \sum_j e_{koj}^2\} - 2(t+1)^{-1} \left[\sum_i \log d_i + \log (1-2\beta_{11})(1-2\beta_{22}) \right] \qquad (3.2.8)$$

$$i = 1,2,\ldots,t; \; j = 1,2,\ldots, K; \; k = 1,2;$$

where $f_1(\underline{\beta}) = (f(\underline{\beta}) - \text{constant})/K(t+1)$,

$$e_{kij} = \{1 - 2(r_{ki}-1)\, r_{ki}^{-1} \beta_{kk}\} Y_{kij} - 2 r_{mi}^{-1} \beta_{km} Y_{mij} - p_{ki} \hat{\mu}_k, \qquad (3.2.9)$$

$m = 2$ when $k = 1$; $m = 1$ when $k = 2$,

$$e_{koj} = (1-2\beta_{kk}) Y_{koj} - \hat{\mu}_k, \qquad (3.2.10)$$

$$d_i = |G_i| = \prod_k \{p_{ki}^{-1} (1-2(r_{ki}-1)\, r_{ki}^{-1} \beta_{kk}\} - 4(r_{1i}\, r_{2i})^{-2} \beta_{12} \beta_{21} \qquad (3.2.11)$$

and $\hat{\mu}_k$ is as in (3.2.5a,b).

The coefficients β's are obtained iteratively. The estimate at the (n+1)th stage of iteration is

$$\underline{\beta}^{(n+1)} = \underline{\beta}^{(n)} - H^{-1} f_1'(\underline{\beta})|_{\underline{\beta}=\underline{\beta}^{(n)}}$$

where $\underline{\beta}' = (\beta_{11}, \beta_{12}, \beta_{22}, \beta_{21}) = (\beta_1, \beta_2, \beta_3, \beta_4)$ say,

$f_1'(\underline{\beta}) = (\partial f_1/\partial\beta_p)$, H is the Hessian matrix i.e. $H = (\partial^2 f_1/\partial\beta_p\partial\beta_q)$; $p, q = 1, 2, 3, 4$.

The elements of $f_1'(\underline{\beta})$ and H are given below.

$\partial f_1/\partial\beta_p$

$$= 2[\sum_i\sum_j e_{kij}(e_{kij})_p' + \sum_j \{e_{koj}(e_{koj})_p'\}](\sum_i\sum_j e_{kij}^2 + \sum_j e_{koj}^2)^{-1}$$

$$-2(t+1)^{-1}\{\sum_i (d_i)_p' d_i^{-1} - 2(1-2\beta_q)^{-1}\},$$

$k = 1$ when $p = 1, 2$; $k = 2$ when $p = 3, 4$, and $\beta_q = \beta_1$ when $p = 1$; $\beta_q = \beta_3$ when $p = 3$, otherwise zero.

$(e_{kij})_p' = \partial e_{kij}/\partial\beta_p$

$$= \begin{cases} \left. \begin{array}{l} -2(r_{ki}-1)r_{ki}^{-1} Y_{kij} + 2K^{-1}p_{ki}(1+\sum_i p_{ki})^{-1} \\ \cdot[\sum_i\sum_j \{(r_{ki}-1)r_{ki}^{-1} Y_{kij}\} + \sum_j Y_{koj}] \end{array} \right\} & \text{For } p = 1,\ k = 1 \\ & \text{For } p = 3,\ k = 2, \\ -2r_{mi}^{-1} Y_{mij} + 2K^{-1}p_{ki}(1 + \sum_i p_{ki})^{-1}\{\sum_i\sum_j r_{mi}^{-1} Y_{mij}\} & \text{For } p=2, k=1 \text{ an } m=2 \\ & \text{For } p=4, k=2 \text{ an } m=1 \end{cases}$$

$(e_{koj})_p' = \partial e_{koj}/\partial\beta_p$ can be obtained from $(e_{kij})_p'$ by replacing

the first term by $-2Y_{koj}$, when $p = 1, 3$ and by zero when $p = 2, 4$.

$$(d_i)'_p = \partial d_i / \partial \beta_p$$

$$= \begin{cases} -2(p_{1i} p_{2i})^{-1} (r_{ki}-1) r_{ki}^{-1} \{1-2(r_{mi}-1) r_{mi}^{-1} \beta_p\} & \text{For } p=1, k=1 \text{ and } m=2 \\ & \text{For } p=3, k=2 \text{ and } m=1 \\ -2(r_{1i} \cdot r_{2i})^{-1} \beta_q & \text{For } p=2, q=4 \\ & \text{For } p=4, q=2 \end{cases}$$

When $p, q = 1, 2$

$$H(p,q) = -4(\sum_i \sum_j e_{1ij}^2 + \sum e_{1oj}^2)^{-2} \{\sum_i \sum_j e_{1ij}(e_{1kj})'_p + (e_{1oj})'_p\} \cdot$$

$$\cdot \{\sum_i \sum_j e_{1oj}(e_{ioj})'_q + \sum_j e_{1oj}(e_{1oj})'_q\} + 2(\sum_i \sum_j e_{1ij}^2 + \sum_j e_{1oj}^2)^{-1} \cdot$$

$$\{\sum_i \sum_j (e_{1ij})'_p (e_{1ij})'_q + \sum_j (e_{1oj})' (e_{1oj})'_q$$

$$+ 2(t+1)^{-1} \sum_i \{(d_i)'_p (d_i)'_q / d_i^2\} + 4 K g(\beta),$$

where $g(\beta) = (1-2\beta_1)^{-1}$ when $p = q = 1$, and zero otherwise.

For $p = q = 3, 4$; e_{1ij} and e_{1oj} are to be replaced by e_{2ij} and e_{2oj}, respectively, in the above expression to get $H(p,q)$.

Here $g(\beta) = (1-2\beta_3)^{-1}$ when $p = q = 3$ and zero otherwise.

When $p = 1, 2$; $q = 3, 4$;

$$H(p,q) = 2(t+1)^{-1} [\sum_i (d_i)'_p (d_i)'_q / d_i^2 + \sum_i ((d_i)'_p)'_q / d_i]$$

where

$$((d_i)'_p)'_q = \partial^2 d_i / \partial\beta_p \partial\beta_q = \begin{cases} 4 \prod_k p_{ki}(r_{ki}-1)\ r^{-1}_{ki} & \text{for } p = 1,\ q = 3 \\ -4(r_{1i}\ r_{2i})^{-2} & \text{for } p = 2,\ q = 4 \\ 0 & \text{otherwise.} \end{cases}$$

emaining elements of H can be obtained from the property of ymmetry.

3 VARIANCE-COVARIANCES OF THE ML ESTIMATORS

The expression for the large sample variance-covariance atrix of the estimators , discussed in section 3.2 , is derived ere. The log-likelihood is

$$\ell = \log L = \text{constant} + K\,[\,\sum_i \log d_i + \log\{(1-2\beta_{11})(1-2\beta_{22})\}\,]$$

$$- \tfrac{1}{2} K(t+1) \log (V_1 . V_2) - \tfrac{1}{2} \sum_i \sum_j (G_i \underline{Y}_{ij} - \mu)' V_i^{-1}(G_i \underline{Y}_{ij} - \underline{\mu})$$

$$- \tfrac{1}{2} \sum_k [\ V_k^{-1} \sum_j \{(1-2\beta_{kk})\ Y_{koj} - \mu_k\}^2\ ], \qquad (3.3.1)$$

ere d_i is given by (3.2.11).

the case of IC let

$$E(\underline{Y}_{ij}) = G_i^{-1}\,\underline{\mu} = \underline{a}_i\ ,\quad V(\underline{Y}_{ij}) = (G_i^{-1}) V_i\ (G_i')^{-1} = E_i\ ,$$

d $E(\underline{Y}_{ij} . \underline{Y}'_{ij}) = \underline{a}_i\ \underline{a}'_i + E_i = F_i$, say ; $i = 1,2,\ldots,t$; $j = 1,2,\ldots, K$;

ere $\underset{(2x2)}{E_i} = (e_{kmi})$, $\underset{(2x2)}{F_i} = (f_{kmi})$, $\underline{a}_i = (a_{1i}, a_{2i})'$. $k,m = 1,2$.

the case of MC let

$$Y_{koj}) = \mu_k(1-2\beta_{kk})^{-1}\ a_{ko}\ ,\quad V(Y_{koj}) = V_k(1-2\beta_{kk})^{-2} = V_{ko}$$

Simplified expressions, after differentiation and taking expectations, are as the following :

$$E(M) = - \begin{bmatrix} K\, V_1^{-1}(\sum_i p_{1i}+1) & 0 \\ 0 & K\, V_2^{-1}(\sum_i p_{2i}+1) \end{bmatrix},$$

$$E(W) = -\frac{1}{2} \begin{bmatrix} K(t+1)V_1^{-1} & 0 \\ 0 & K(t+1)V_2^{-1} \end{bmatrix}, \quad E(A) = 0$$

and $E(Y_{koj}^2) = a_{ko}^2 + V_{ko} = f_{ko}$, say.

The large sample variance-covariance matrix of the ML estimators of $\underline{\mu}$, $\underline{\nu}$ and $\underline{\beta}$ is given by

$$\operatorname{Var}\begin{bmatrix}\hat{\underline{\mu}}\\ \hat{\underline{\nu}}\\ \hat{\underline{\beta}}\end{bmatrix} = -\left[E\left\{\begin{matrix} M & A & T\\ A' & W & Z\\ T' & Z' & B\end{matrix}\right\}\right]^{-1}$$

where $\underline{\nu} = (V_1, V_2)'$, $\underset{(2\times2)}{M} = (\partial^2 £ / \partial^2 \mu_k \partial \mu_m)$, $\underset{(2\times2)}{W} = (\partial^2 £ / \partial V_k \partial V_m)$,

$\underset{(4\times4)}{B} = (\partial^2 £ / \partial\beta_p \partial\beta_q)$, $\underset{(2\times2)}{A} = (\partial^2 £ / \partial\mu_k \partial V_m)$, $\underset{(2\times2)}{T} = (\partial^2 £ / \partial\mu_k \partial\beta_p)$,

$\underset{(2\times4)}{Z} = (\partial^2 £ / \partial V_k \partial\beta_p)$, $k, m = 1, 2$; $p, q = 1, 2, 3, 4$.

The elements of the symmetric matrix B are as the following :

For $p = 1, 3$

$E(B(p,p))$

$$= -K\Big[\sum_i \{ (d_i)'_p \, d_i^{-1} \}^2 - 4(1-2\beta_p)^{-2} - 4V_k^{-1} \sum_i p_{ki}^{-1}(r_{ki}-1) r_{ki}^{-1} f_{kki} + f_{ko} \Big],$$

$E(B(p,p+1))$

$$= -K\Big[\sum_i \{ (d_i)'_p (d_i)'_{p+1} \, d_i^{-2} \} - 4V_k^{-1} \sum_i \{ p_{ki}^{-1}(r_{1i} \cdot r_{2i})^{-1} (r_{ki}-1) \, f_{12i} \} \Big].$$

$k = 1$ when $p = 1$; $k = 2$ when $p = 3$.

The expected values of $B(1,4)$ and $B(1,3)$ are given by

$$E(B(p,q)) = -K\Big[\sum_i \{ (d_i)'_p (d_i)'_q \, d_i^{-2} \} + \sum_i \{ ((d_i)'_p)'_q \, d_i^{-1} \} \Big].$$

$$E(T) = -2\begin{bmatrix} V_1^{-1}K\{\sum_i (r_{1i}-1)r^{-1}a_{1i}+a_{1o}\} & V_1^{-1}K\sum_i r_{2i}^{-1}a_{2i} & 0 & 0 \\ 0 & 0 & V_2^{-1}K\{\sum_i (r_{2i}-1)r_{2i}^{-1}a_{2i}+a_{2o}\} & V_2^{-1}K\sum_i r_{1i}^{-1}a_{1i} \end{bmatrix}$$

$$E(z) = \begin{bmatrix} z_{11} & z_{12} & 0 & 0 \\ 0 & 0 & z_{21} & z_{22} \end{bmatrix}$$

$$z_{11} = -2KV_1^{-2}\left[\sum_i p_{1i}^{-1}(r_{1i}-1)r_{1i}^{-1}\{1-2(r_{1i}-1)r_{1i}^{-1}\beta_{11}\} f_{11i}-2r_{2i}^{-1}\beta_{12}f_{12i}-p_{1i}\mu_1 a_{1i} \right]$$

$$z_{12} = -2KV_1^{-2}\left[\sum_i p_{1i}^{-1} r_{2i}^{-1}\{1-2(r_{1i}-1)r_{1i}^{-1}\beta_{11}\} f_{12i}-2r_{2i}^{-1}\beta_{12}f_{22i}-p_{1i}\mu_1 a_{2i} \right]$$

$$z_{22} = -KV_2^{-2}\left[\sum_i p_{2i}^{-1}(r_{2i}-1)r_{2i}^{-1}\{1-2(r_{2i}-1)r_{2i}^{-1}\beta_{22}\} f_{22i}-2r_{1i}^{-1}\beta_{21}f_{21i}-p_{2i}\mu_2 a_{2i} \right]$$

$$z_{21} = -KV_2^{-2}\left[\sum_i p_{2i}^{-1} r_{2i}^{-1}\{1-2(r_{2i}-1)r_{2i}^{-1}\beta_{11}\} f_{21i}-2r_{1i}^{-1}\beta_{21}f_{11i}-p_{2i}\mu_2 a_{1i} \right]$$

The expected values B(1,4) and B(2,3) are given by

$$E(B(p,q) = -K \left[\sum_i \{ (d_i)'_p \, (d_i)'_q \, d_i^{-2} \} \right].$$

For p = 2,4

$$E(B(p,p)) = -K\left[\sum_i \{ (d_i)'_p \, d_i^{-1} \}^2 - 4V_k^{-1} \sum_i p_{ki}^{-1} \; r_{mi}^{-2} \; f_{mmi} \right.$$

For p = 2, k = 1, and m = 2 ; for p = 4, k = 2 and m = 1.

3.4 FITTING THE MODEL WITH BLOCK EFFECTS

In agricultural field experiments blocking will usually eliminate large scale fertility differences. By considering the arrangement of the rows in randomized blocks, the simultaneous equations after introducing the block parameters are

$$Y_{1ij} = p_{1i}(\mu_1+\alpha_{1j}) + 2(r_{1i}-1)r_{1i}^{-1} \beta_{11} Y_{1ij} + 2r_{2i}^{-1} \beta_{12} Y_{2ij} + \varepsilon_{1ij} \tag{3.4.1a}$$

$$Y_{2ij} = p_{2i}(\mu_2+\alpha_{2j}) + 2(r_{2i}-1)r_{2i}^{-1} \beta_{22} Y_{2ij} + 2r_{1i}^{-1} \beta_{21} Y_{1ij} + \varepsilon_{2ij} , \tag{3.4.1b}$$

where $\alpha_{1j}(1-2\beta_{11})^{-1}$ and $\alpha_{2j}(1-2\beta_{22})^{-1}$ are the effects of the jth block on the first and second crops. The expressions that estimates μ_1 and μ_2 in (3.2.5a,b) will estimate $\mu_1+\bar{\alpha}_1$ and $\mu_2+\bar{\alpha}_2$, where $\bar{\alpha}_k = \sum_j \alpha_{kj}/K$, k = 1,2 . The estimating equations for error variances V_1 and V_2 are

$$-2r_{2i}^{-1}\,\beta_{12}\,Y_{2ij}\,\} + (1-2\beta_{11})\,Y_{1oj}\,, \qquad (3.4.3a)$$

$$(1 + \sum_i p_{2i})\,(\widehat{\mu_2+\alpha_{2j}}) = \sum_i \; (1-2(r_{2i}-1)\,r_{2i}^{-1}\,\beta_{22})\;Y_{2ij}$$

$$-2r_{1i}^{-1}\,\beta_{21}\,Y_{1ij}\,\} + (1-2\beta_{22})\;Y_{2oj}\,. \qquad (3.4.3b)$$

The function to be minimized for obtaining the ML estimates of the competition coefficients is same as (3.2.7) except $\hat{V}_1$ and $\hat{V}_2$ are to be obtained by (3.4.2a,b) . The equation to be used iteratively for obtaining the ML estimates , discussed in section 3.2.2 , remains same except for μ_k. In (3.2.9) and (3.2.10) $\hat{\mu}_k$ is to be replaced by $\widehat{\mu_k+\alpha_{kj}}$ as given in (3.4.3a,b).

$$K(t+1)\,\hat{V}_1 = \sum_i \sum_j p_{1i}^{-1} \{(1-2(r_{1i}-1)\, r_{1i}^{-1}\, \beta_{11})\, Y_{1ij} -2r_{2i}^{-1}\, \beta_{12}\, Y_{2ij} - p_{1i}(\widehat{\mu_1 + \alpha_{1j}})\}^2 + \sum_j \{(1-2\beta_{11})Y_{1oj} - (\widehat{\mu_1+\alpha_{1j}})\}^2 \,. \tag{3.4.2a}$$

$$K(t+1)\,\hat{V}_2 = \sum_i \sum_j p_{2i}^{-1} \{(1-2(r_{2i}-1)\, r_{2i}^{-1}\, \beta_{11})\, Y_{2ij} -2r_{1i}^{-1}\, \beta_{21}\, Y_{1ij} - p_{2i}(\widehat{\mu_2+\alpha_{2j}})\}^2 + \sum_j \{(1-2\beta_{22})Y_{2oj} - (\widehat{\mu_2+\alpha_{2j}})\}^2 \,. \tag{3.4.2b}$$

where

$$(1 + \sum_i p_{1i})\,(\widehat{\mu_i+\alpha_{ij}}) = \sum_i \{(1-2(r_{1i}-1)\, r_{1i}^{-1}\beta_{11})Y_{1ij}$$

3.5 MONTE CARLO SIMULATION STUDY

A simulation study is carried out to study the performance of the ML estimates of the parameters for an experiment of the moderate size. The Monte Carlo experiment consists of 36 plots arranged in three randomized blocks. Among 12 plots in each block 10 plots are with row arrangements of the first and the second component crops as 1:1, 1:2, 1:3, 2:1, 2:2, 2:3, 3:1, 3:2, 4:4 and the remaining two plots are of MC. Values of the parameters in MC are taken as $\mu_1 = 60$, $\mu_2 = 50$ and $\beta_{11} = \beta_{22} = -0.4$. The values chosen of the parameters are such that the expected yields are in the range of what we find in sorghum and pearlmillet MC. Three situations of interspecific competition are considered by choosing β_{12} and β_{21} as
i) (-0.1, -0.1), (ii) (-0.1, -0.4), and (iii) (-0.1,-0.6), respectively.

Data for 500 experiments are simulated with V_1 and V_2 such that the coefficient of variation (CV) are 10% and 20% for two MC's, respectively, which is the usual range to be observed in the field experiments. The number 500 is chosen as the changes in the average values of the estimates are considerably smaller for higher numbers. The IMSL subroutine (GGNRM) package is used to generate the pseudo bivariate normal variates. Block effects are assumed to be zero. The ML estimates of β's have been obtained by solving the nonlinear equations, using Newton - Raphson iterative method described in section 3.2, by including the block parameters. The starting values are obtained

by simplex subroutine (Nelder and Mead, 1965) of NAG. The results for three different cases considered are given in Table 3.1. The ML estimates obtained by the Newton-Raphson method are close to the starting values given by the simplex subroutine. The convergence of iterative process did not create any problem except being slow in about one percent of the cases. Convergence is said to be achieved when the absolute values of the partial derivatives of the β coefficients are less than 10^{-3} and the successive values of $f_1(\underline{\beta})$ had a relative change of less than 10^{-5}. The average number of iterations required to converge is found to be approximately three.

Relative bias and MSE for the estimates are calculated as

$$\text{bias}\ (\beta_{km}) = (\hat{\beta}_{km.} - \beta_{km}) , \tag{3.5.1}$$

$$\text{relative bias}\ (\hat{\beta}_{km}) = 100 \times \text{bias}\ (\hat{\beta}_{km}) \ / \ \beta_{km} , \tag{3.5.2}$$

where $\hat{\beta}_{km.} = \sum_{i=1}^{R} \hat{\beta}_{kmi} \ / \ R , \ R = 500$;

$$\text{MSE}(\hat{\beta}_{km}) = \sum_{i=1}^{R} (\hat{\beta}_{kmi} - \beta_{km})^2 \ / \ R. \tag{3.5.3}$$

The values of bias and $\sqrt{\text{MSE}}$ for the three interspecific competition situations at CV 10% and 20% are given in Table 3.1 . From the results (Table 3.1) it is evident that the bias and the $\sqrt{\text{MSE}}$ are dependent on CV and also on the values of the parameters to some extent . This is expected from the expressions of large sample variance-variance matrix in section 3.3. Bias and $\sqrt{\text{MSE}}$ are considerably large at 20%

Table 3.1 Results of simulation study for three sets of parameters values at 10% and 20% CV.

Set No.		β_{11}		β_{12}		β_{22}		β_{21}		μ_1		μ_2		V_1		V_2	
	C V%	10	20	10	20	10	20	10	20	10	20	10	20	10	20	10	20
1.	Parameter Values	-0.40	-0.40	-0.10	-0.10	-0.40	-0.40	-0.10	-0.10	60.00	60.00	50.00	50.00	36.00	144.00	25.00	100.00
	Biasx10^2	-1.51	-2.54	-1.41	-2.00	-0.91	-2.50	-0.89	-2.10	99.79	161.28	60.69	162.87	-454.31	-1538.05	-289.74	-872.97
	$\sqrt{MSE}$x10^1	1.60	2.25	1.55	2.19	1.70	2.30	1.06	1.53	112.87	155.78	98.13	136.00	112.68	507.19	76.70	348.15
2.	Parameter Values	-0.40	-0.40	-0.10	-0.10	-0.40	-0.40	-0.40	-0.40	60.00	60.00	50.00	50.00	36.00	144.00	25.00	100.00
	Biasx10^2	0.62	0.60	1.43	1.81	-1.73	-2.92	-1.25	-2.32	56.36	-42.54	95.18	168.19	533.50	-1878.91	-273.34	-905.15
	$\sqrt{MSE}$x10^1	0.75	1.31	1.18	1.64	1.78	2.41	0.99	1.46	46.85	72.67	90.32	125.77	103.66	463.35	76.35	347.89
3.	Parameter Values	-0.40	-0.40	-0.10	-0.10	-0.40	-0.40	-0.60	-0.60	60.00	60.00	50.00	50.00	36.00	144.00	25.00	100.00
	Biasx10^2	-0.16	-0.14	1.52	2.75	-1.78	-2.90	-0.62	-2.17	-4.14	30.33	71.97	163.36	-432.19	-1549.66	-284.69	-905.82
	$\sqrt{MSE}$x10^1	0.66	1.32	0.68	1.26	1.42	2.15	0.70	1.21	31.59	63.73	63.94	102.27	106.97	501.14	74.81	338.57

than that at 10%, as expected. The parameters μ_1 and μ_2 are estimated with a relative bias of not more than 3% . In the case of competition parameters the relative bias varied upto 15% when CV is 10% and to 27% when CV is 20% . In most of the cases bias is less than onetenth of $\sqrt{MSE}$. In all the cases the bias contributed only a small proportion of the MSE. For most of the cases the variance of the estimates is at least 99% of the MSE. In general it appears from this simulation study that the parameters can be estimated satisfactory at 10% CV except when the competition coefficients are very small. When the competition coefficients are small then the relative mean square is large, as expected. However, in such cases relative mean square error is not a very good measure of the variability.

The effect of increasing the number of replications to six and nine has been examined. The results are given in Table 3.2 for one set of parameter values at 10% and 20% CV. The bias decreases when the number of replications are increased and the variances decrease proportionately with the increase in the number of replications. This reduction is expected. The maximum relative bias reduced from 14% to about 10% and 9% at 10% CV, and from 21% to about 16% and 10% at 20% CV when the replications are increased from three to six and nine, respectively. This suggests that when variability is large and competition coefficients are small, large number of replications are needed to estimate the parameters precisely.

$\beta_{11} = \beta_{22} = -0.40, \ \beta_{12} = \beta_{21} = -0.10,$

CV		β_{11}	β_{12}	β_{22}
				Replications =
10 %	Bias x 10^2	-1.51	-1.41	-0.91
	$\sqrt{MSE}$ x 10^1	1.60	1.55	1.70
				Replications =
	Bias x 10^2	-0.20	0.16	-1.33
	$\sqrt{MSE}$ x 10^1	1.12	1.08	1.25
				Replications =
	Bias x 10^2	-1.17	-0.89	-0.81
	$\sqrt{MSE}$ x 10^1	0.96	0.91	0.96
20 %				Replications =
	Bias x 10^2	-2.54	-2.00	-2.50
	$\sqrt{MSE}$ x 10^1	2.25	2.19	2.30
				Replications =
	Bias x 10^2	-1.10	-0.07	-2.45
	$\sqrt{MSE}$ x 10^1	1.50	1.47	1.58
				Replications =
	Bias x 10^2	-1.69	-1.00	-1.82
	$\sqrt{MSE}$ x 10^1	1.23	1.18	1.26

$\mu_1 = 60,\ \mu_2 = 50,\ CV = 10\%$ and 20% .

β_{21}	μ_1	μ_2
3		
-0.89	99.79	60.69
1.06	112.87	98.13
6		
-0.95	0.34	83.42
0.77	73.05	71.13
9		
-0.40	70.68	42.71
0.58	66.43	54.13
3		
-2.10	161.28	162.87
1.53	155.78	136.00
6		
-1.62	48.45	152.41
1.06	103.54	95.81
9		
-0.08	94.67	94.89
0.79	84.36	71.40

There is a close agreement between the large sample variance-covariance matrix obtained through the expressions, derived in section 3.3, and empirical variance-covariances obtained through simulation study. For example, the variance-covariance matrix from simulation study, Emp. $V(\hat{\underline{\beta}})$, and the asymptotic variance-covariance matrix, Asy. $V(\hat{\underline{\beta}})$, for the competition parameters are given below for the parameter values,

$\beta_{11} = \beta_{22} = -0.40$, $\beta_{12} = \beta_{21} = -0.10$, $\mu_1 = 60.00$, $\mu_2 = 50.00$ and CV = 10% .

$$\text{Emp. } V(\hat{\underline{\beta}}) = 10^{-2} \begin{bmatrix} 2.57 & 2.28 & -0.90 & -0.61 \\ & 2.39 & -0.97 & -0.65 \\ & & 2.87 & 1.67 \\ & & & 1.13 \end{bmatrix} \tag{3.5.4}$$

$$\text{Asy. } V(\hat{\underline{\beta}}) = 10^{-2} \begin{bmatrix} 2.46 & 2.11 & -0.78 & -0.52 \\ & 2.19 & -0.82 & -0.56 \\ & & 2.68 & 1.49 \\ & & & 0.99 \end{bmatrix} \tag{3.5.5}$$

where $\underline{\beta} = (\beta_{11}, \beta_{12}, \beta_{22}, \beta_{21})'$.

A similar type of agreement is observed for the other cases.

3.6 DISCUSSION

In the absence of exact tests, we shall use tests based on the asymptotic variances and likelihood ratio, which are only approximate for finite sample sizes. The ML estimators obtained here are consistent but this gives no idea of the magnitude

of bias which are likely to occur for any given size of the experiment. The simulation study carried out here gives some idea about the magnitude of bias and approximations involved in using large sample expressions for variance-covariance matrix.

In general there exists competition among the crops, consequently the competition coefficients are taken negative in simulation study. These coefficients are selected in such a way that the expected yields are realistic. Willey (1979) describes three different situations in IC. He named them as i) 'mutual cooperation' if the intraspecific competition of both the species is greater than their interspecific competition, ii) 'mutual inhibition' if the intraspecific competition of both the species is less than their interspecific competition, and iii) 'mutual compensation' if the intraspecific competition of one of the species is greater than its interspecific competition, and for the other species the former is less than the latter. The first type of competition is not unusual. The second type is rare and in this case IC is clearly not advantageous over MC. The third type of situation is most common. Among the three cases selected for simulation the first one represents mutual cooperation, and the second and third represent mutual compensation.

The choice of the initial values for starting the iterative procedure, discussed in section 3.2.2, is important otherwise

there will be a problem in the convergence. The simplex procedure adopted gave satisfactory values of the estimates; the Newton-Raphson method further improved these estimates to satisfy the required convergence criterion. The possibility of convergence to a local maxima has been examined in detail for some cases by giving different initial values, but the function always converged to the same value. This rules out the possibility of convergence to a local maxima.

It is evident from the values of MSE that more number of replications are required when variability is large or the competition parameters are small in magnitude. One can search for optimum designs such that the parameters can be estimated precisely through an experiment of reasonable size .

If the individual row yields are available it would be interesting to examine the gain in efficiency in estimating the parameters. However, this will involve a lot more work in data collection and computation. In IC experiment it is a normal practice to record plot yields and consequently this has not been pursued.

CHAPTER IV

SPATIAL MODEL WITH VARYING POPULATION DENSITIES

4.1 INTRODUCTION

Plant population of the component crops is another major factor that affects the total competition pressure per unit area , as has been pointed out in the first chapter. The knowledge about the nature of response to varying plant densities will help in arriving at the optimum density levels for achieving maximum yield. The relationship of yield and plant populations in IC needs a detailed investigation as in the case of MC (Willey, 1979). In MC Willey and Heath (1969) reviewed the practical aspects of the relationship between plant density (plant population per unit area) and crop yield. The inverse power response curve suggested by Bleasdale and Nelder (1960) is the one which is widely used. Mead (1970) examined the theoretical properties of this relationship and also discussed estimation procedures. Gillies and Rotkowsky (1978) found that the maximum likelihood estimates are biased and to overcome this partially they suggested a reparameterization. The plant density relationship based on the principles of competition in MC is discussed by Watkinson (1980). This is again a reparameterized form of the Bleasdale and Nelder equation. The parameters in this equation describe the biological process of the competition and hence ensure wider applicability. The equation developed in this chapter for IC

turns out to be a natural extension of this relation.

The model suggested in the second chapter has been extended to include the effect of plant population at a constant row width. The estimation procedure for this extended model is discussed for field experiments in great detail. A particular case of the model in terms of plant densities only is also considered.

4.2 DEVELOPMENT OF MODEL

In section 2.4 we have seen in the case of MC the classical yield density relation

$$E(Y) = N\delta + \beta\rho\, E(Y) , \tag{4.2.1}$$

is derived by giving equal weightage to the neighbours which lie within a fixed distance. Now we examine the effect by varying interplant distances. As the distance between the neighbouring plants increases the competition decreases. Let us assume that the relationship between competition coefficient β and the interplant distance x be given by

$$\beta(x) = \beta_o\, x^{-c} , \tag{4.2.2}$$

where β_o and c are constants. There is an empirical evidence for this relationship (Vandermeer, 1986). Let the interrow distance , x_o , be constant , then

$$\rho = (x_o\, x)^{-1} , \text{ i.e. } \rho \propto x^{-1} , \tag{4.2.3}$$

and $(x_o x)$ is the area occupied by a single plant.

From (4.2.1), (4.2.2) and (4.2.3)

$$E(Y) = N\delta + \beta_o \rho^{c_1} E(Y),$$

where $c_1 = c+1$. The expected value of the yield per plant $\omega (= Y/N)$, is given by

$$E(\omega) = \delta(1 - \beta_o \rho^{c_1})^{-1} = (a + b \rho^{c_1})^{-1}, \qquad (4.2.4)$$

where $a = \delta^{-1}$ and $b = -\beta_o/\delta$.

The general form of the yield - density relation suggested by Bleasdale and Nelder is

$$E(1/\omega^{\theta}) = \alpha + \beta\rho^{\varphi}. \qquad (4.2.5)$$

The ratio θ/φ determines the shape of the curve. It is asymptotic if $\theta/\varphi \geq 1$, otherwise there exists a maximum at a finite value of ρ. Based on this fact Bleasdale (1966) suggested $\varphi = 1$, which belongs to the family of curves given by (4.2.5). Apart from the approximation $E(1/\omega) \simeq 1/E(\omega)$, the equation (4.2.4) is a particular case of (4.2.5) when $\theta = 1$. Approximation may be justified by ignoring the higher order terms and noting that ω takes only positive values. Hence the model developed above, based on the principles of plant competition in MC, belongs to the widely accepted Bleasdale and Nelder equation. The equation developed in this chapter for IC is a natural extension of (4.2.4).

Development of model for 2 : 2 row arrangement

We shall start with the competition model at the individual plant level in IC. From this, a model for the plot yields, including the plant density effects, will be developed.

For simplicity consider 2 : 2 row arrangement. The assumptions regarding inter and intrarow spacings in IC and MC are the

same as discussed in section 2.2. Let y_{11} and y_{12} be the yields of rows 1 and 2 of the first species, y_{21} and y_{22} be the yields of the two rows for the second species, ω_{1ki} and ω_{2mj} be the yields of ith and jth plants in kth and mth rows of first and second species, respectively, $(k,m = 1,2)$. We assume here that the yield of any plant within a row is affected by its immediate neighbouring plants apart from the immediate neighbouring rows. Let there be n_k plants in a row of the kth species. If the competition coefficient due to the neighbouring row is β_{km}, then on the average each plant experiances a competition of β_{km}/n_k due to the neighbouring row. Let ω_{11i} and ω_{21j} be the yields of the ith an jth plant of the first rows of crop 1 and 2, respectively, then,

$$\omega_{11i} = \delta_1 + \beta_1'(\omega_{11,i-1} + \omega_{11,i+1}) + \beta_{11}'y_{12}/n_1 + \beta_{12}'y_{22}/n_1 + e_{11i} \tag{4.2.6a}$$

$$\omega_{21j} = \delta_2 + \beta_2'(\omega_{12,j-1} + \omega_{12,j+1}) + \beta_{22}'y_{22}/n_2 + \beta_{21}'y_{12}/n_2 + e_{21j}, \tag{4.2.6b}$$

$$i = 1,2,\dots n_1;\ j = 1,2,\dots,n_2;$$

where δ_1 and δ_2 are the expected plant yields in the absence of intra and interrow competitions; e_{11i}, e_{21j} are the errors assumed to be distributed independently as $N(0,e_1^2)$ and $N(0,e_2^2)$, respectively.

By adding the equations corresponding to all ω_{1li} and ω_{2lj}, we have

$$y_{11} = n_1\delta_1 + 2\beta'_1 y_{11} + \beta'_{11} y_{12} + \beta'_{12} y_{22} + u'_{11}, \quad (4.2.7a)$$

$$y_{21} = n_1\delta_2 + 2\beta'_2 y_{21} + \beta'_{22} y_{22} + \beta'_{21} y_{12} + u'_{21}, \quad (4.2.7b)$$

where $\quad y_{kl} = \sum_{i=1}^{n_k} \omega_{kli}, \quad u'_{kl} = \sum_{i=1}^{n_k} e_{kli}.$

The above equations can be written as

$$y_{11} = \eta_1 + \beta_{11} y_{12} + \beta_{12} y_{22} + u_{11}, \quad (4.2.8a)$$

$$y_{21} = \eta_2 + \beta_{22} y_{22} + \beta_{21} y_{12} + u_{21}, \quad (4.2.8b)$$

where $\eta_k = n_k \delta_k (1-\beta'_k)^{-1}$, $\beta_{km} = \beta'_{km}(1-\beta'_k)^{-1}$, $u_{kl} = u'_{kl}(1-\beta'_k)^{-1}$.

These equations agree with the equations formulated earlier for row arrangements (2.2.1). Now let us examine the effect of varying number of plants within a row by keeping the interrow distances fixed. From (4.2.7a,b) the average per plant yield of the first and second crop can be expressed as

$$\bar{\omega}_{11} = \delta_1 + 2\beta'_1 \bar{\omega}_{11} + \beta'_{11} \bar{\omega}_{12} + \beta'_{12} n_2 n_1^{-1} \bar{\omega}_{22} + \bar{u}'_{11}, \quad (4.2.9a)$$

$$\bar{\omega}_{21} = \delta_2 + 2\beta'_2 \bar{\omega}_{21} + \beta'_{22} \bar{\omega}_{22} + \beta'_{21} n_1 n_2^{-1} \bar{\omega}_{12} + \bar{u}'_{21}, \quad (4.2.9b)$$

where $\bar{\omega}_{km} = y_{km}/n_k$; $\bar{u}'_{kl} = u'_{kl}/n_k$.

For a fixed row length the number of plants within a row is proportional to the average distance between the plants. Let x_1 and x_2 be the average plant to plant distances within rows of the first and second species, respectively, and x_o be the fixed interrow distance. From equations (4.2.2) and (4.2.9a,b), we have,

$$\bar{\omega}_{11} = \delta_1 + 2\beta'_{10}\, x_1^{-c_1}\, \bar{\omega}_{11} + \beta'_{110}\, x_1^{-c_{11}}\, \bar{\omega}_{12} + n_2\, n_1^{-1}\, \beta'_{120}\, x_2^{-c_{12}}\, \bar{\omega}_{22} + \bar{u}'_{11},$$

(4.2.10a)

$$\bar{\omega}_{21} = \delta_2 + 2\beta'_{20}\, x_2^{-c_2}\, \bar{\omega}_{21} + \beta'_{220}\, x_2^{-c_{22}}\, \bar{\omega}_{22} + n_1\, n_2^{-1}\, \beta'_{21}\, x_1^{-c_{21}}\, \bar{\omega}_{12} + \bar{u}'_{12},$$

(4.2.10b)

where c_k, c_{km}, $k,m = 1,2$ are some constants.

Since the interrow distances are fixed

$$x_k^{-1} \propto (x_o\, x_k)^{-1} = \rho_k\, ,$$

where ρ_k is plant density of the kth species.

The above equations can be written in terms of plant densities as

$$\bar{\omega}_{11} = \delta_1 + 2\beta'_{10}\rho_1^{c_1}\, \bar{\omega}_{11} + \beta'_{110}\, \rho_1^{c_{11}}\, \bar{\omega}_{12} + n_2\, n_1^{-1}\, \beta'_{120}\, \rho_2^{c_{12}}\, \bar{\omega}_{22} + \bar{u}'_{11},$$

(4.2.11a)

$$\bar{\omega}_{21} = \delta_2 + 2\beta'_{20}\, \rho_2^{c_2}\, \bar{\omega}_{21} + \beta'_{220}\, \rho_2^{c_{22}}\, \bar{\omega}_{22} + n_1\, n_2^{-1}\, \beta'_{210}\, \rho_1^{c_{21}}\, \bar{\omega}_{12} + \bar{u}'_{21}.$$

(4.2.11b)

The equations in terms of row yields are

$$y_{11} = n_1\delta_1 + 2\beta'_{10}\rho_1^{c_1} y_{11} + \beta'_{110} \rho_1^{c_{11}} y_{12} + \beta'_{120} \rho_2^{c_{12}} y_{22} + u'_{11} , \quad (4.2.12a)$$

$$y_{21} = n_2\delta_2 + 2\beta'_{20} \rho_2^{c_2} y_{21} + \beta'_{220} \rho_2^{c_{22}} y_{22} + \beta'_{210} \rho_1^{c_{21}} y_{12} + u'_{21} . \quad (4.2.12b)$$

Similarly we can write the equations for the other two rows as

$$y_{12} = n_1\delta_1 + 2\beta'_{10} \rho_1^{c_1} y_{12} + \beta'_{110} \rho_1^{c_{11}} y_{11} + \beta'_{120} \rho_2^{c_{12}} y_{21} + u'_{12} , \quad (4.2.13a)$$

$$y_{22} = n_2\delta_2 + 2\beta'_{20} \rho_2^{c_2} y_{22} + \beta'_{220} \rho_2^{c_{22}} y_{21} + \beta'_{210} \rho_1^{c_{21}} y_{11} + u'_{22} . \quad (4.2.13b)$$

From the above , equations in terms of total yields are

$$Y_1 = 2n_1\delta_1 + 2\beta'_{10} \rho_1^{c_1} Y_1 + \beta'_{110} \rho_1^{c_{11}} Y_1 + \beta'_{120} \rho_2^{c_{12}} Y_2 + \varepsilon_1 , \quad (4.2.14a)$$

$$Y_2 = 2n_2\delta_2 + 2\beta'_{20} \rho_2^{c_2} Y_2 + \beta'_{220} \rho_2^{c_{22}} Y_2 + \beta'_{210} \rho_1^{c_{21}} Y_1 + \varepsilon_2 , \quad (4.2.14b)$$

where $Y_k = y_{k1} + y_{k2}$, $\varepsilon_k = u'_{k1} + u'_{k2}$.

Model for monocropping

In MC the jth plant yield in the ith row of the kth species , ω_{koij} , can be expressed as

$$Y_{koi} = n_k \delta_k + 2\beta'_{ko} \rho_k^{c_k} Y_{koi} + \beta'_{kko} \rho_k^{c_{kk}} (Y_{ko,i-1} + Y_{ko,i+1}) + u'_{koi} . \tag{4.2.17}$$

For the total yield of four rows, we have

$$Y_{ko} = 4n_k \delta_k + 2\beta'_{ko} \rho_k^{c_k} Y_{ko} + 2\beta'_{kko} \rho_k^{c_{kk}} Y_{ko} + \varepsilon_{ko} , \tag{4.2.18}$$

where $Y_{ko} = \sum_{i=1}^{4} Y_{koi}$, $\varepsilon_{ko} = \sum_{i=1}^{4} u'_{koi}$.

$$E(Y_{ko}) = 4n_k \delta_k (1 - 2\beta'_{ko} \rho_k^{c_k} Y_{ko} - 2\beta'_{kko} \rho_k^{c_{kk}})^{-1} ,$$

and

$$\begin{aligned} E(\bar{\omega}_{ko}) &= \delta_k (1 - 2\beta'_{ko} \rho_k^{c_k} Y_{ko} - 2\beta'_{kko} \rho_k^{c_{kk}})^{-1} \\ &= \delta_k (1 - b_{ko} \rho_k^{c_k}) , \text{ if } c_k = c_{kk} , \end{aligned} \tag{4.2.19}$$

where $b_{ko} = 2(\beta'_{ko} + \beta'_{kko})$.

$$\omega_{koij} = \delta_k + \beta'_k(\omega_{koi,j-1} + \omega_{koi,j+1}) + \beta'_{kk}(y_{ko,i-1} + y_{ko,i+1})/n_k + e_{koij} ,$$

$$k = 1,2; \quad i = 1,2,3,4; \quad j = 1,2,\ldots,n_k . \qquad (4.2.15)$$

n terms of row yield, we have

$$y_{koi} = n_k \delta_k + 2\beta'_k y_{koi} + \beta'_{kk}(y_{ko,i-1} + y_{ko,i+1}) + u'_{koi} , \qquad (4.2.16)$$

here $\quad y_{koi} = \sum_{j=1}^{n_k} \omega_{koij} , \quad u'_{koi} = \sum_{j=1}^{n_k} e_{koij} .$

y varying the intrarow distances and using the same assumptions s in IC , we have the model in terms of plant density as

The above equation is same as (4.2.4). For simplicity we reparameterize δ_k and ρ_k as

$$4n_k\ \delta_k = 4n_k\ n_{ko}^{-1}\ \ n_{ko}\ \delta_k = n_k\ n_{ko}^{-1}\ \varphi_{ko} = \rho_k\ \rho_{ko}^{-1}\ \ \varphi_{ko} = d_k\ \varphi_{ko}\ ,$$

where $\varphi_{ko} = 4n_{ko}\ \delta_k$, $d_k = \rho_k\ \rho_{ko}^{-1}$, ρ_{ko} is plant density when there are n_{ko} plants per unit length of crop k.

Hence (4.2.18) can be written as

$$Y_{ko} = d_k\ \ \varphi_{ko} + 2\beta_{ko}\ d_k^{c_k}\ Y_{ko} + 2\beta_{kko}\ d_k^{c_{kk}}\ Y_{ko} + \varepsilon_{ko}\ , \qquad (4.2.20)$$

where $\quad \beta_{ko} = \beta'_k\ \rho_{ko}^{c_k}\ ,\ \beta_{kko} = \ \beta'_{kko}\ \rho_{ko}^{c_{kk}}\ .$

Generalization to $r_1 : r_2$ arrangement in IC

From (2.2. 8) and (4.2.14a,b) the equations for the total yield of r_1 rows of the first crop and r_2 rows of the second crop are

$$Y_1 = r_1\ n_1\ \delta_1 + 2\beta'_{10}\ \rho_1^{c_1}\ Y_1 + 2(r_1-1)\,r_1^{-1}\ \beta'_{110}\ \rho_1^{c_{11}}\ Y_1$$

$$+\ 2r_2^{-1}\ \beta'_{120}\ \rho_2^{c_{12}}\ Y_2 + \varepsilon_1 \qquad , \qquad (4.2.21a)$$

$$Y_2 = r_2\ n_2\ \delta_2 + 2\beta'_{20}\ \rho_2^{c_2}\ Y_2 + 2(r_2-1)\,r_2^{-1}\ \beta'_{220}\ \rho_2^{c_{22}}\ Y_1$$

$$+\ 2r_1^{-1}\ \beta'_{210}\ \rho_1^{c_{21}}\ Y_1 + \varepsilon_2\ . \qquad (4.2.21b)$$

For the case of MC we have

$$Y_{ko} = (r_1+r_2)\,n_k\ \delta_k + 2\beta'_{ko}\ \rho_k^{c_k}\ Y_{ko} + 2\beta'_{kko}\ \rho_k^{c_{kk}}\ Y_{ko} + \varepsilon_{ko}\ .$$

can write this equation as (4.2.20) , by taking

$$\varphi_{ko} = (r_1+r_2)\, n_{ko}\, \delta_k \,.$$

e equations (4.2.21a,b) can be written as

$$Y_1 = p_1\, d_1\, \varphi_{10} + 2\beta_{10}\, d_1^{c_1}\, Y_1 + 2(r_1-1)r_1^{-1}\, \beta_{110}\, d_1^{c_{11}}\, Y_1$$

$$+ 2r_2^{-1}\, \beta_{120}\, d_2^{c_{12}}\, Y_2 + \varepsilon_1 \,, \qquad (4.2.22a)$$

$$Y_2 = p_2\, d_2\, \varphi_{20} + 2\beta_{20}\, d_2^{c_2}\, Y_2 + 2(r_2-1)r_2^{-1}\, \beta_{220}\, d_2^{c_{22}}\, Y_2$$

$$+ 2r_1^{-1}\, \beta_{210}\, d_1^{c_{21}}\, Y_1 + \varepsilon_2 \,, \qquad (4.2.22b)$$

here $\beta_{kmo} = \rho_{ko}^{c_k}$ and $p_k = r_k / (r_1+r_2)$.

quations (4.2.22a,b) can be written as

$$\{1-2\beta_{10} d_1^{c_1} - 2(r_1-1)\, r_1^{-1}\, \beta_{110}\, d_1^{c_1}\}\, Y_1 = p_1\, d_1\, \varphi_{10} + 2r_2^{-1}\, \beta_{120}\, d_2^{c_{12}}\, Y_2 + \varepsilon_1 ,$$

$$(4.2.23a)$$

$$\{1-2\beta_{20}\, d_2^{c_2} + 2(r_2-1)\, r_2^{-1}\, \beta_{220}\, d_2^{c_2}\}\, Y_2 = p_2\, d_2\, \varphi_{20} + 2r_1^{-1}\, \beta_{210}\, d_1^{c_{21}}\, Y_1 + \varepsilon_2$$

$$(4.2.23b)$$

urther this can be expressed in matrix notation as

$$(I-H)\underline{Y} = P\, D\, \underline{\varphi}_o + \underline{\varepsilon} \,, \qquad (4.2.24)$$

here $\underline{Y} = (Y_1, Y_2)'$, $\underline{\varphi}_o = (\varphi_{10}, \varphi_{20})'$, $\underline{\varepsilon} = (\varepsilon_1, \varepsilon_2)'$, $D = \mathrm{diag}\,(d_1, d_2)$,

$$(I-H) = \begin{bmatrix} 1-2\beta_{10} d_1^{c_1} - 2(r_1-1) r_1^{-1} \beta_{110} d_1^{c_{11}} & -2r_2^{-1} \beta_{120} d_2^{c_{12}} \\ -2r_1^{-1} \beta_{210} d_1^{c_{21}} & 1-2\beta_{20} d_2^{c_2} - 2(r_2-1) r_2^{-1} \beta_{220} d_2^{c_{22}} \end{bmatrix}$$

In MC

$$(1 - 2\beta_{ko} d_k^{c_k} - 2\beta_{kko} d_k^{c_{kk}}) Y_{ko} = d_k \varphi_{ko} + \varepsilon_{ko} , \quad (4.2.25)$$

where $$\varepsilon_{ko} = \sum_{i=1}^{r_1+r_2} \sum_{j=1}^{n_k} e_{kij} .$$

$$V(\varepsilon_{ko}) = (r_1+r_2) n_k e_k^2$$

$$= d_k V_{ko} ,$$

where $V_{ko} = (r_1+r_2) n_{ko} e_k^2$. We assume e_{koij}'s are independently and normally distributed with mean zero and variance e_k^2. From the above assumptions Y_{ko} follows a normal distribution with

$$E(Y_{ko}) = (1 - 2\beta_{ko} d_k^{c_k} - 2\beta_{kko} d_k^{c_{kk}})^{-1} d_k \varphi_{ko} ,$$

$$(4.2.26)$$

$$V(Y_{ko}) = (1 - 2\beta_{ko} d_k^{c_k} - 2\beta_{kko} d_k^{c_{kk}})^{-2} d_k V_{ko} .$$

In the case of IC $\underline{Y}$ follows a bivariate normal distribution with

$$E(\underline{Y}) = (I-H)^{-1} P D \underline{\varphi}_o ,$$

$$(4.2.27)$$

$$V(\underline{Y}) = (I-H)^{-1} P^{1/2} D^{1/2} V_o P^{1/2} D^{1/2} (I-H')^{-1} .$$

The extension to a plot containing N_1 rows of first crop and N_2 rows of second crop is straightforward and is on similar lines as discussed in section 3.2.

4.3 ESTIMATION OF THE PARAMETERS

Consider the experiment discussed in section 3.2 where variation in plant densities has been introduced by changing plant to plant distances within rows. Let d_{1m_1}, d_{2m_2}, $m_1 = 1, 2, \ldots, s_1$; $m_2 = 1, 2, \ldots, s_2$ be the plant density levels in MC for the two species, whereas in IC they are d_{1m}, d_{2m}, $m = 1, 2, \ldots, s$. The total number of experimental plots are $K(s_1 + s_2 + st)$, where K is the number of replications. From the equations (4.2.23a,b) individual plot yields of the component crops in IC can be expressed by the following bivariate sets of simultaneous stochastic equations :

$$(1 - 2\beta_{10}\, d_{1m}^{c_1} - 2(r_{1i}-1)\, r_{1i}^{-1}\, \beta_{110}\, d_{1m}^{c_{11}})\, Y_{1imj} = P_{1i}\, d_{1m}\, \varphi_{10}$$

$$+ 2r_{2i}^{-1}\, \beta_{120}\, d_{2m}^{c_{12}}\, Y_{2imj} + \varepsilon_{1imj}\,, \qquad (4.3.1a)$$

$$(1 - 2\beta_{20}\, d_{2m}^{c_2} - 2(r_{2i}-1)\, r_{2i}^{-1}\, \beta_{220}\, d_{2m}^{c_{22}})\, Y_{2imj} = P_{2i}\, d_{2m}\, \varphi_{20}$$

$$+ 2r_{1i}^{-1}\, \beta_{210}\, d_{1m}^{c_{21}}\, Y_{1imj} + \varepsilon_{2imj}\,, \qquad (4.3.1b)$$

$$i = 1, 2, \ldots, t\,;\; m = 1, 2, \ldots, s\,;\; j = 1, 2, \ldots, k.$$

This can be written as

$$(1-H_{im})\, \underline{Y}_{imj} = P_i\, D_m\, \underline{\varphi}_o + \underline{\varepsilon}_{imj}\,, \qquad (4.3.2)$$

where

In the case of MC the equations for individual plot yields from (4.2.25) are

$$(1 - 2\beta_{ko}\, d_{km_k}^{c_k} - 2\beta_{kko}\, d_{km_k}^{c_{kk}})\, Y_{kom_kj} = d_{km_k}\,\varphi_{ko} + \varepsilon_{kom_kj} \,. \qquad (4.3.4)$$

Y_{kom_kj} follows a normal distribution with

$$E(Y_{kom_kj}) = b_{kom_k}^{-1}\, d_{km_k}\,\varphi_{ko} \qquad (4.3.5)$$

$$V(Y_{kom_kj}) = b_{kom_k}^{-2}\, d_{km_k}\, V_{ko}$$

where $b_{kom_k} = (1 - \beta_{ko}\, d_{km_k}^{c_k} - 2\rho_{kko}\, d_{km_k}^{c_{kk}})$, $k = 1,2$. (4.3.6)

$$H_{im} = \begin{bmatrix} 1-2\beta_{10}\, d_{1m}^{c_1} - 2(r_{1i}-1)\, r_{1i}^{-1}\, \beta_{110}\, d_{1m}^{c_{22}} & -2r_{2i}^{-1}\, \beta_{120}\, d_{2m}^{c_{12}} \\ -2r_{1i}^{-1}\, \beta_{210}\, d_{1m}^{c_{21}} & 1-2\beta_{20}\, d_{2m}^{c_2} - 2(r_{2i}-1)\, r_{2i}^{-1}\, \beta_{220}\, d_{2m}^{c_{22}} \end{bmatrix},$$

$P_i = \mathrm{diag}\,(p_{1i}, p_{2i})$, $D_m = \mathrm{diag}\,(d_{1m}, d_{2m})$,

$\underline{\varphi}_o = (\ _{10},\ _{20})'$, $\underline{\varepsilon}_{imj} = (\varepsilon_{1imj}, \varepsilon_{2imj})'$.

Under the assumptions made on errors $\underline{Y}_{imj}$ follows a bivariate normal distribution with

$$E(\underline{Y}_{imj}) = (I-H_{im})^{-1}\, P_i\, D_{im}\, \underline{\varphi}_o ,$$

$$V(\underline{Y}_{imj}) = (I-H_{im})^{-1}\, P_i^{1/2}\, D_m^{1/2}\, V_o\, P_i^{1/2}\, D_m^{1/2}\, (I - H'_{im})^{-1}, \qquad (4.3.3)$$

where $D_m^{1/2} = \mathrm{diag}(\sqrt{d_{1m}}, \sqrt{d_{2m}})$.

$-2\log L$ for all the observations in MC and IC is given by

$$-2\log L = \text{constant} - 2K \sum_j \sum_m \log |I - H_{im}| + \sum_k \sum_{m_k} \log b_{kom_k}$$
$$+ K(s_1+st) \log V_{10} + K(s_2+st) \log V_{20}$$
$$+ \sum_i \sum_m \sum_j (G_{im} \underline{Y}_{imj} - \underline{\varphi}_o)' V_{oim}^{-1} (G_{im} \underline{Y}_{imj} - \underline{\varphi}_o)$$
$$+ \sum_k \sum_{m_k} \sum_j d_{km_k}^{-1} V_{ko}^{-1} (b_{kom_k} Y_{kom_k j} - d_{km_k} \varphi_{ko})^2 , \tag{4.3.7}$$

where

$$G_{im} = P_i^{-1} D_m^{-1} (I-H_{im}), \quad V_{oim} = V_o P_i^{-1} D_m^{-1} .$$

Assuming $\underline{\beta} = (\beta_{10}, \beta_{110}, \beta_{120}, \beta_{20}, \beta_{220}, \beta_{210})'$ as known and equating to zero the partial differentials with respect to φ_{10}, φ_{20}, V_{10} and V_{20}, the estimating equations of these parameters are the following :

$$K(\sum_{m_1} d_{1m_1} + \sum_m p_{1i} d_{1m}) \hat{\varphi}_{10} = \sum_i \sum_m \sum_j (b_{11im} Y_{1imj} + b_{12im} Y_{2imj})$$
$$+ \sum_{m_1} \sum_j b_{10m_1} Y_{10m_1 j} , \tag{4.3.8a}$$

$$K(\sum_{m_2} d_{2m_2} + \sum_m p_{2i} d_{2m}) \hat{\varphi}_{20} = \sum_i \sum_m \sum_j (b_{22im} Y_{2imj} + b_{21im} Y_{1imj})$$
$$+ \sum_{m_2} \sum_j b_{20m_2} Y_{20m_2 j} , \tag{4.3.8b}$$

$$K(s_1+st)\hat{V}_{10} = \sum_i \sum_m \sum_j p_{1i}^{-1} d_{1m}^{-1} (b_{11im} Y_{1imj} + b_{12im} Y_{2imj} - p_{1i} d_{1m} \hat{\varphi}_{10})^2$$
$$+ \sum_{m_1} \sum_j (b_{10m_1} Y_{10m_1 j} - d_{1m_1} \hat{\varphi}_{10})^2 \tag{4.3.9a}$$

and

$$K(s_2+st)\,\hat{V}_{20} = \sum_i \sum_m \sum_j p_{2i}^{-1}\, d_{2m}^{-1}\, (b_{22im}\, Y_{2imj} + b_{21im}\, Y_{1imj} - p_{2i}\, d_{2m}\, \hat{\varphi}_{20})^2$$

$$+ \sum_{m_2} \sum_j (b_{20m_2}\, Y_{20m_2 j} - d_{2m_2}\, \hat{\varphi}_{20})^2 \,, \qquad (4.3.9b)$$

where

$$\begin{aligned} b_{kkim} &= 1 - 2\beta_{ko}\, d_{km}^{c_k} - 2(r_{ki}-1)\, r_{ki}^{-1}\, \beta_{kko}\, d_{km}^{c_{kk}} \,, \\ b_{12im} &= -2r_{2i}^{-1}\, \beta_{120}\, d_{2m}^{c_{12}} \,, \qquad (4.3.10) \\ b_{21im} &= -2r_{1i}^{-1}\, \beta_{210}\, d_{1m}^{c_{21}} \,. \end{aligned}$$

Substituting these values in the log - likelihood function (4.3.7) and simplifying , leads to the function

$$f(\underset{\sim}{\beta}) = \text{constant} - 2K\Big[\sum_i \sum_m \log |I-H_{im}| + \sum_k \sum_j \log b_{koj}\Big]$$

$$+ K(s_1+st)\log \hat{V}_{10} + K(s_2+st)\log \hat{V}_{20} \,, \qquad (4.3.11)$$

where

$$\underset{\sim}{\beta} = (\beta_{10}\,,\ \beta_{110}\,,\ \beta_{120}\,,\ \beta_{20}\,,\ \beta_{220}\,,\ \beta_{210})'$$

$$= (\beta_1\,,\ \beta_2\,,\ \beta_3\,,\ \beta_4\,,\ \beta_5\,,\ \beta_6)' \,, \text{ say} \,.$$

The ML estimate of $\underset{\sim}{\beta}$ is the value corresponding to the minimum of $f(\underset{\sim}{\beta})$.

4.4 FITTING THE MODEL WITH BLOCK EFFECTS

By considering the arrangement of the plots in randomized blocks, the equations (4.3.1a,b) get modified as,

$$b_{11im}\, Y_{1imj} = p_{1i}\, d_{1m}\, (\varphi_{10} + \alpha_{1j}) + b_{12im}\, Y_{2imj} + \varepsilon_{1imj}\,, \tag{4.4.1a}$$

$$b_{22im}\, Y_{2imj} = p_{2i}\, d_{2m}(\varphi_{20}+\alpha_{2j}) + b_{21im}\, Y_{1imj} + \varepsilon_{2imj}\,. \tag{4.4.1b}$$

In the case of MC

$$b_{kom_k}\, Y_{km_kj} = d_{km_k}\, (\varphi_{ko} + \alpha_{kj}) + \varepsilon_{kom_kj}\,, \tag{4.4.2}$$

where $\alpha_{kj}\, b^{-1}_{kom_kj}$ is the effect of the jth block on crop k.

The log - likelihood function is same as (4.3.7) except that φ_{ko} is replaced by φ_{koj}, where $\varphi_{koj} = (\varphi_{ko} + \alpha_{kj})$. The expressions in (4.3.8a,b) for estimation of φ_{ko} now estimate $\varphi_{ko} + \bar{\alpha}$, where $\bar{\alpha}_k = \sum_j \alpha_{kj}/K$.

The estimating equations for V_{10} and V_{20} are

$$K(s_1+st)\, \hat{V}_{10} = \sum_i \sum_m \sum_j p^{-1}_{1i}\, d^{-1}_{1m}\, (b_{11im}\, Y_{1imj} + b_{12im}\, Y_{2imj} - p_{1i}\, d_{1m}\, \hat{\varphi}_{10j})^2 + \sum_{m_1} \sum_j d^{-1}_{1m_1}(b_{10}\, Y_{10m_1j} - d_{1m_1}\hat{\varphi}_{10j})^2 \tag{4.4.3a}$$

$$K(s_2+st)\ \hat{V}_{20} = \sum_i \sum_m \sum_j p_{2i}^{-1}\ d_{2m}^{-1}\ (b_{22im}\ Y_{2imj} + b_{21im}\ Y_{1imj}$$

$$- p_{2i}\ d_{2m}\ \hat{\varphi}_{20j})^2 + \sum_{m_2} \sum_j d^{-1}_{2m_2}(b_{20}\ Y_{20m_2j} - d_{2m_2}\ \hat{\varphi}_{20j})^2,$$

(4.4.3b)

where,

$$(\sum_{m_1} d_{2m_1} + \sum_i \sum_m p_{1i}\ d_{1m})\hat{\varphi}_{10j} = \sum_i \sum_m (b_{11im}\ Y_{1imj} + b_{12im}\ Y_{2imj})$$

$$+ \sum_{m_1} b_{10m_1}\ Y_{10m_1}\ , \qquad (4.4.4a)$$

$$(\sum_{m_2} d_{2m_2} + \sum_i \sum_m p_{2i}\ d_{2m})\hat{\varphi}_{20j} = \sum_i \sum_m (b_{22im}\ Y_{2imj} + b_{21im}\ Y_{1imj})$$

$$+ \sum_{m_2} b_{20m_2}\ Y_{20m_2}\ . \qquad (4.4.4b)$$

The function to be minimized for estimating $\underline{\beta}$ is same as (4.3.11) with $\hat{V}_{10}$ and $\hat{V}_{20}$ as in (4.4.3a,b).

4.5 MONTE CARLO SIMULATION STUDY

A simulation study is carried out to examine the performance of ML estimators , in the extended model including block parameters. The Monte Carlo experiment consists of both IC and MC plots. In IC three levels of each factor are selected. They are $r_{1i}:r_{2i}$ = 1:1 , 1:2 , 2:1 ; d_{1m} = 1,2,3 and d_{2m} = 1,2,3. All the 27 combinations of the levels of these factors are considered . In MC there are three density levels for each crop and they are d_{1m_1} = 1,2,3 and d_{2m_2} = 1,2,3 . Randomized block design with

three replications is used. Hence the experiment consists of 99 plots arranged in three complete blocks.

The values selected for the intraspecific competition coefficients are

$$\beta_{10} = \beta_{20} = -0.40 \ , \ \beta_{110} = \beta_{220} = -0.30, \ \varphi_{10} = 60.00, \varphi_{20} = 40.00 .$$

In IC the interspecific competition coefficients are selected as

$$\beta_{120} = \beta_{210} = -0.10 \ .$$

Values for the error variances V_{10} and V_{20} are chosen such that CV is 15% in MC. We assume $c_1 = c_{11} = c_{21}$ and $c_2 = c_{22} = c_{12}$. The values considered for these constants are

$$c_1 = c_{11} = c_{21} = c_2 = c_{22} = c_{12} = 1, 2 \ .$$

In each case of $c_k = 1$ and $c_k = 2$ data for 500 experiments are generated by assuming block effects as zeroes. The parameters are estimated by using the iterative equations given in appendix 4.1. The initial value are obtained by Nelder and Mead simplex algorithm. The convergence criterion adopted is same as in section 3.5 . The values of bias (3.5.1) and $\sqrt{MSE}$ (3.5.3) are given in Table 4.1 . For the competition coefficients , $\underline{\beta}$, φ_{10} and φ_{20} , relative biases do not exceed 2.5% in both the cases. The relative $\sqrt{MSE}$ to its population values do not exceed 27% when $c_1 = c_2 = 1$, and 14% when $c_1 = c_2 = 2$. In general the

β_{10}	-0.40	-1.01	0.83	0.10	0.47
β_{110}	-0.30	-0.70	0.55	-0.24	0.42
β_{120}	-0.10	-0.10	0.06	0.11	0.13
β_{20}	-0.40	-0.88	1.08	-0.58	0.52
β_{220}	-0.30	-0.66	0.62	-0.10	0.39
β_{210}	-0.10	-0.18	0.17	-0.20	0.10
φ_{10}	60.00	105.13	79.41	-1.45	52.98
φ_{20}	40.00	51.69	68.68	45.72	39.71
V_{10}	81.00	-205.22	225.36	-405.63	165.23
V_{20}	36.00	16.62	101.25	-67.87	71.86

Table 4.1 Bias and $\sqrt{MSE}$ of ML estimates of $\underline{\beta}$ based on simulation study

		$c_1=c_{11}=c_{21}=1$ $c_2=c_{22}=c_{12}=1$		$c_1=c_{11}=c_{21}=2$ $c_2=c_{22}=c_{21}=2$	
Parameter	Values	Bias x 10^2	$\sqrt{MSE}$ x 10^1	Bias x 10^2	$\sqrt{MSE}$ x 10^1

parameters are estimated precisely. However, the size of the experiment is considerably large. One can also see the precision of the estimates by their asymptotic variance-covariances. For example, the asymptotic variance - covariance matrix for the competition coefficients Var $(\underline{\beta})$, when $c_1 = c_2 = 2$, is given below :

$$\mathrm{Var}(\hat{\underline{\beta}}) = 10^{-4} \begin{bmatrix} 15.62 & 9.16 & 5.72 & -0.03 & -0.09 & -0.05 \\ & 17.42 & 6.76 & -0.05 & -0.15 & -0.08 \\ & & 6.64 & -0.06 & -0.17 & -0.09 \\ & & & 19.37 & 10.82 & 4.30 \\ & & & & 19.53 & 4.25 \\ & & & & & 1.87 \end{bmatrix}, \quad (4.5.1)$$

where $\underline{\beta} = (\beta_{10}, \beta_{110}, \beta_{120}, \beta_{20}, \beta_{220}, \beta_{210})'$,

$\beta_{10} = \beta_{20} = -0.40$, $\beta_{110} = \beta_{220} - -0.30$, $\beta_{120} = \beta_{210} = -0.10$,

$\varphi_{10} = 60.00$, $\varphi_{20} = 40.00$, $V_{10} = 81.00$, $V_{20} = 36.00$.

4.6 A PARTICULAR CASE OF INTEREST

4.6.1 Model in terms of plant densities only

Investigation of row arrangements and plant density levels simultaneously requires an experiment of large size. In many IC experimental programmes studies on the effect of variation due to plant density levels, at a fixed row arrangements, are not uncommon. A model to analyse

the experimental data from this type of experiment can be easily derived from (4.2.22a,b). Under the assumption $c_1 = c_{11} = c_{21}$ and $c_2 = c_2 = c_{12}$, we have

$$(1 - 2\beta_{10} d_1^{c_1} - 2(r_1-1) r_1^{-1} \beta_{110} d_1^{c_1}) Y_1 = d_1 \varphi_{10} + 2r_2^{-1} \beta_{120} d_2^{c_2} Y_2 + \varepsilon_1; \tag{4.6.1a}$$

$$(1 - 2\beta_{20} d_2^{c_2} - 2(r_2-1) r_2^{-1} \beta_{220} d_2^{c_2}) Y_2 = d_2 \varphi_{20} + 2r_1^{-1} \beta_{210} d_1^{c_1} Y_1 + \varepsilon_2. \tag{4.6.1b}$$

These equations can be written as

$$(1 - \beta_1' d_1^{c_1}) Y_1 = d_1 \varphi_1 + \beta_{12}' d_2^{c_2} Y_2 + \varepsilon_1, \tag{4.6.2a}$$

$$(1 - \beta_2' d_2^{c_2}) Y_2 = d_2 \varphi_2 + \beta_{21}' d_1^{c_1} Y_1 + \varepsilon_2, \tag{4.6.2b}$$

where

$$\beta_1' = 2\beta_{10} + 2(r_1-1) r_1^{-1} \beta_{110}, \quad \beta_2' = 2\beta_{20} + 2(r_2-1) r_2^{-1} \beta_{220},$$

$$\beta_{mk}' = 2r_k^{-1} \beta_{mko}, \quad \varphi_k = p_k \varphi_{ko}, \quad \varepsilon_k = \sum_i \sum_j u_{kij},$$

$$k = 1,2;\ i = 1,2,\ldots, N_k;\ j = 1,2,\ldots, n_k.$$

In the matrix form the equations can be written as,

$$(I-H)\ \underline{Y} = D\ \underline{\varphi} + \underline{\varepsilon}, \tag{4.6.3}$$

where

$$(I-H) = \begin{bmatrix} 1 - \beta_1' \, d_1^{c_1} & - \beta_{12}' \, d_2^{c_2} \\ -\beta_{21}' \, d_1^{c_1} & 1 - \beta_2' \, d_2^{c_2} \end{bmatrix},$$

$\underline{\varphi} = (\varphi_1, \varphi_2)'$, $\underline{Y} = (Y_1, _2)'$ and $\underline{\varepsilon} = (\varepsilon_1, \varepsilon_2)'$.

The mean and variance are

$$E(\underline{Y}) = (I-H)^{-1} D \underline{\varphi} \quad \text{and } V(\underline{Y}) = (I-H)^{-1} D^{1/2} V D^{1/2} (I-H')^{-1} , \tag{4.6.4}$$

where $V = \text{diag}\,(V_1, V_2)$, $V_k = p_k V_{10}$.

4.6.2 Estimation of parameters

Let us consider an experiment with s density levels of both the crops arranged in K randomized blocks. The individual plot yields can be represented as ,

$$(1 - \beta_1 d_{1i}^{c_1}) Y_{1i} = d_{1i} \varphi_1 + \beta_{12} d_{2i}^{c_2} Y_{2ij} + \varepsilon_{1ij}, \tag{4.6.5a}$$

$$(1 - \beta_2 d_{2i}^{c_2}) Y_{2i} = d_{2i} \varphi_2 + \beta_{21} d_{1i}^{c_1} Y_{1ij} + \varepsilon_{2ij}, \tag{4.6.5b}$$

$$i = 1, 2, \ldots, s \; ; \; j = 1, 2, \ldots, K.$$

In the matrix notation these equations are ,

$$(I-H_i) \underline{Y}_{ij} = D_i \underline{\varphi} + \underline{\varepsilon}_{ij} , \tag{4.6.6}$$

where

$$(I-H_i) = \begin{bmatrix} 1 - \beta_1' \, d_{1i}^{c_1} & -\beta_{12}' \, d_{2i}^{c_2} \\ -\beta_{21}' \, d_{1i}^{c_1} & 1-\beta_2' \, d_{2i}^{c_2} \end{bmatrix},$$

$\underline{\varphi} = (\varphi_1, \varphi_2)'$, $\underline{Y}_{ij} = (Y_{1ij}, Y_{2ij})'$ and $\underline{\varepsilon}_{ij} = (\varepsilon_{1ij}, \varepsilon_{2ij})'$.

The equation (4.6.5), after including the block parameters, can be written as

$$(I-H_i)\ \underline{Y}_{ij} = D_i\ \underline{\varphi}_j + \underline{\varepsilon}_{ij}\ , \tag{4.6.7}$$

where $\underline{\varphi}_j = (\underline{\varphi} + \underline{\alpha}_j)$, $\underline{\alpha}_j = (\alpha_{1j}, \alpha_{2j})'$.

The value of -2log L is

$$-2\log L = \text{constant} - 2K \sum_i \log |I-H_i| + K\ s \log(V_1 \cdot V_2)$$

$$+ \sum_i \sum_j \{ d_{1i}^{-1}\ V_1\ \varepsilon_{1ij}^2 + d_{2i}^{-1}\ V_2\ \varepsilon_{2ij}^2 \}. \tag{4.6.8}$$

Following the same procedure as earlier , we obtain

$$\hat{\varphi}_1 = (K + \sum_i d_{1i})^{-1} \sum_i \sum_j \{(1 - \beta_1'\ d_{1i}^{c_1})\ Y_{1ij} - \beta_{12}'\ d_{2i}^{c_2}\ \ Y_{2ij}\}$$

$$\hat{\varphi}_2 = (K + \Sigma\ d_{2i})^{-1} \sum_i \sum_j \{(1 - \beta_2'\ d_{2i}^{c_2})\ Y_{2ij} - \beta_{21}'\ d_{1i}^{c_1}\ Y_{1ij}\ \}$$

$$\hat{V}_k = (K \cdot s)^{-1} \sum_i \sum_j\ d_{1i}\ e_{kij}^2\ ,\ k = 1, 2\ ;$$

where

$$e_{1ij} = (1 - \beta_1'\ d_{1i}^{c_1})\ Y_{1ij} - \beta_{12}'\ d_{2i}^{c_2}\ \ Y_{2ij} - d_{1i}\ \hat{\varphi}_{1j}$$

$$e_{2ij} = (1 - \beta_2'\ d_{2i}^{c_2})\ Y_{2ij} - \beta_{21}'\ d_{1i}^{c_1}\ Y_{1ij} - d_{2i}\ \hat{\varphi}_{2j}$$

$$\hat{\varphi}_{1j} = (\Sigma\ d_{1i})^{-1} \sum_i\ \{(1 - \beta_1' d_{1i}^{c_1})\ Y_{1ij} - \beta_{12}'\ d_{2i}^{c_2}\ Y_{2ij}\}$$

$$\hat{\varphi}_{2j} = (\sum_i d_{2i})^{-1} \sum_i\ \{\ (1 - \beta_2'\ d_{2i}^{c_2})\ Y_{2ij} - \beta_{21}'\ d_{1i}^{c_1}\ Y_{1ij}\} .$$

The function to be minimized for estimation of β's is

$$f_1(\underline{\beta}) = \log\{\sum_i \sum_j d_{1i}^{-1} e_{1ij}^2\} + \log\{\sum_i \sum_j d_{2i}^{-1} e_{2ij}^2\} - 2s^{-1} \sum_i \log D_i ,$$

where

$$D_i = |I-H_i| = (1 - \beta_1' d_{1i}^{c_1})(1 - \beta_2' d_{2i}^{c_2}) - \beta_{12}' \beta_{21}' d_{1i}^{c_1} d_{2i}^{c_2} . \tag{4.6.9}$$

4.7 DISCUSSION

The model suggested by Wright (1.2.3a,b) for plant density levels can be obtained by replacing Y_2 by Y_1 and Y_1 by Y_2, respectively, in the right hand side of the equations (4.6.2a,b), as follows:

$$(1 - \beta_1' d_1^{c_1} - \beta_{12}' d_2^{c_2}) Y_1 = d_1 \varphi_1 + \varepsilon_1 ,$$

$$(1 - \beta_2' d_1^{c_1} - \beta_{21}' d_1^{c_1}) Y_2 = d_2 \varphi_2 + \varepsilon_2 .$$

Now taking $c_1 = c_2 = 1$, one obtains

$$E(Y_1) = d_1 \varphi_1 (1 - \beta_1' d_1 - \beta_{12}' d_2)^{-1}$$

$$E(Y_2) = d_2 \varphi_2 (1 - \beta_2' d_1 - \beta_{21}' d_1)^{-1} ,$$

which are same as that of Wright. However, there is a difference in the error structure. The assumptions made in extending the model for IC to incorporate the variations due to plant densities in this chapter are logical extensions of MC situation.

For keeping discussions general different constants c_1, c_{11}, c_{12}, c_2, c_{22} and c_{21} are introduced in the model.

But , in practice, the constants associated with the density d_k may not differ. Hence we can assume $c_1 = c_{11} = c_{21}$ and $c_2 = c_{22} = c_{12}$. In the MC situation (4.2.25) c_k and c_{kk} usually lie between 1 and 2. Consequently c_k and c_{kk} are taken as 1 and 2 in simulation study. In practice the problem of estimation of c_1 and c_2 also arises. It has been observed that due to high correlations among the estimates of c_1 , c_2 and competition parameters there is a considerable problem in the convergence of the iterative process.

The model developed in section 4.2 involves six competition parameters. Use of a good numerical procedure becomes very important when the dimension of competition parameters is large. It is noticed in some cases that the iterating procedure using Newton-Raphson equations converge to some local maximum , which is far off from the values given by the simplex procedure. In such cases quasi-Newton iterative procedure, which reduces the step length , is required to achieve the convergence near the values given by the simplex procedure. This procedure was applied to obtain the ML estimates when such a problem arose.

In general there is a good agreement between the estimates of variances obtained by asymptotic expressions (appendix 4.2) and those obtained empirically through simulation.

For example, when the parameter values are

$\beta_{10} = \beta_{20} = -0.40$, $\beta_{110} = \beta_{220} = -0.30$, $\beta_{120} = \beta_{210} = -0.10$

$\varphi_{10} = 60.00$, $\varphi_{20} = 40.00$, $V_{10} = 81.00$, $V_{20} = 40.00$,

$c_1 = c_2 = 2$,

the empirical estimates and estimates using asymptotic expressions are the following :

Estimated Variances of the Parameter $\times 10^2$

	β_{10}	β_{110}	β_{120}	β_{20}	β_{220}	β_{210}
Empirical	0.22	0.17	0.02	0.27	0.15	0.01
Asymptotic	0.16	0.17	0.07	0.19	0.20	0.02

From the simulation results, and also from the asymptotic variance - covariances, it is evident that the parameters can be estimated precisely for the type of experiment considered in section 4.5 at 15% CV. However, the sizes of the experiment and blocks are considerably large. It is, therefore, important to search for certain optimum designs such that the parameters can be estimated precisely through an experiment of reasonable size.

APPENDIX 4.1

A NUMERICAL METHOD FOR FINDING THE MAXIMUM LIKELIHOOD ESTIMATES

For simplicity, taking $s_1 = s_2$ the ML estimate of $\underline{\beta}$ correspond to the minimum of the function

$$f_1(\underline{\beta}) = \log \Big(\sum_i \sum_m \sum_j p_{1i}^{-1} d_{1m}^{-1} e_{1mj}^2 + \sum_{m_1} \sum_j d_{1m_1}^{-1} e_{10m_1 j}^2 \Big)$$

$$+ \log \Big(\sum_i \sum_m \sum_j p_{2i}^{-1} d_{2m}^{-1} e_{2mj}^2 + \sum_{m_2} \sum_j d_{2m_2}^{-1} e_{20m_2 j}^2 \Big)$$

$$-2 \Big(\sum_i \sum_m \log D_{im} + \sum_k \sum_{m_k} \log D_{kom_k} \Big) / g \, , \qquad (A4.1.1)$$

where

$f_1(\underline{\beta}) = (f(\underline{\beta}) - \text{constant}) \, K^{-1} g^{-1}$,

$g = s_1 + st$,

$f(\underline{\beta})$ is as given in (4.3.11),

$e_{1imj} = b_{11im} Y_{1imj} + b_{12im} Y_{2imj} - p_{1i} d_{1m} \hat{\varphi}_{10j}$,

$e_{2imj} = b_{22im} Y_{2imj} + b_{21im} Y_{1imj} - p_{2i} d_{2m} \hat{\varphi}_{20j}$,

$e_{10m_1 j} = b_{10m_1} Y_{10m_1 j} - d_{1m_1} \hat{\varphi}_{10j}$,

$e_{20m_2 j} = b_{20m_2} Y_{20m_2 j} - d_{2m_2} \hat{\varphi}_{20j}$,

$D_{im} = |I - H_{im}| = b_{11im} b_{22im} - b_{12im} b_{21im}$,

b_{11im}, b_{22im}, b_{12im} and b_{21im} are as in (4.3.10).

$$+ \sum_{m_k} \sum_{j} d^{-1}_{km_k} e_{kom_kj} (e_{kom_kj})'_p \}$$

$$- \{ \sum_{i} \sum_{m} (D_{im})'_p D^{-1}_{im} + \sum_{m_k} b_{ko} b^{-1}_{kom_k} \} / g] , \quad \text{(A4.1.3)}$$

$k = 1$ when $p = 1, 2, 3$; $k = 2$ when $p = 4, 5, 6$. This also holds true for the subsequent expressions. In the above

$$a_{kk} = \sum_{i} \sum_{m} \sum_{j} p^{-1}_{ki} d^{-1}_{km} e^{2}_{kimj} , \quad a_{ko} = \sum_{m_k} \sum_{j} d^{-1}_{km_k} e^{2}_{kom_kj} ,$$

$$(e_{kimj})'_p = \partial e_{kimj} / \partial \beta_p$$

$$= (b_{kkim})'_p Y_{kimj} - p_{ki} d_{km} (\hat{\varphi}_{koj})'_p , \quad \text{when } p = 1, 2, 4, 5 ;$$

$$= (b_{knim})'_p Y_{nimj} - p_{kimj} d_{km} (\hat{\varphi}_{koj})'_p , \quad \text{when } p = 3, 6 ;$$

b_{kom_k} is as in (4.3.6) and $\underline{\beta} = (\beta_{10}, \beta_{110}, \beta_{120}, \beta_{20}, \beta_{220}, \beta_{210})'$

$= (\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6)'$, say.

$\underline{\beta}$ is to be obtained iteratively by taking the estimate at the (n+1)th stage as

$$\underline{\beta}^{(n+1)} = \underline{\beta}^{(n)} - H^{-1} f_1'(\underline{\beta})_{\underline{\beta} = \underline{\beta}^{(n)}} , \qquad (A4.1.2)$$

where

$$f_1'(\underline{\beta}) = (\partial f / \partial \beta_p) \text{ and } H = (\partial^2 f / \partial \beta_p \partial \beta_q), \quad p, q = 1, 2, \ldots, 6.$$

The elements of $f'(\underline{\beta})$ are

$$\partial f_1 / \partial \beta_p = 2[(a_{kk} + a_{ko})^{-1} \{\sum_i \sum_j \sum_m p_{ki} \, d_{km}^{-1} \, e_{kijm} (e_{kijm})_p',$$

$$(\varphi_{koj})'_p = \Big[\sum_i \sum_m \sum_j (b_{kkim})'_p \, Y_{kimj} + \sum_{m_k} (b_{kom_k})'_p \, Y_{kom_k j} \Big] / h_k \qquad \text{when } p = 1, 2, 4, 5$$

$$= \Big[\sum_i \sum_m \sum_j (b_{knim})'_p \, Y_{nimj} \Big] / h_k \, , \qquad \text{when } p = 3, 6 \, ;$$

where

$$h_k = \Big(\sum_{m_k} d_{km_k} + \sum_i \sum_m p_{ki} \, d_{km} \Big) \, ,$$

and

$$(D_{im})'_p = D_{im} / \partial \beta_p = (b_{kkim})'_p \, b_{nnim} \, , \qquad \text{when } p = 1, 2, 4, 5$$

$$= -(b_{knim})'_p \, b_{knim} \, , \qquad \text{when } p = 3, 6 \, .$$

= 2 when k = 1 ; n = 1 when k = 2 and the same holds in the ubsequent expressions. In the above

$$(b_{kkim})'_p = \partial b_{kkim} / \beta_p \begin{cases} = -2 d_{km}^{c_k} & , \quad \text{when } p = 1, 4 \\ = -2(r_{ki}-1)\, r_{ki}^{-1}\, d_{km}^{c_{kk}} & , \quad \text{when } p = 2, 5 \\ = -2\, r_{ni}^{-1}\, d_{nm}^{c_{kn}} & , \quad \text{when } p = 3, 6 \\ = 0 & , \quad \text{otherwise} \end{cases}$$

$$(b_{kom_k})'_p = b_{kom_k} / \beta_p \begin{cases} = -2 d_{km_k}^{c_k} & , \quad \text{when } p = 1, 4 \\ = -2 d_{km_k}^{c_{kk}} & , \quad \text{when } p = 2, 5 \\ = 0 & , \quad \text{otherwise ,} \end{cases}$$

$$+ \sum_{m_k} \sum_{j} d_{koj}^{-1} (e_{kom_kj})'_p (e_{kom_kj})'_q \}$$

$$+ \{ \sum_{i} \sum_{m} (D_{im})'_p (D_{im})'_q D_{im}^{-2} \} / g$$

$$+ \sum_{m_k} (b_{kom_k})'_p (b_{kom_k})'_q b_{kom_k}^{-2} .$$

For $p = 1, 2, 3$ and $q = 4, 5, 6$;

$$\partial^2 f_1 / \partial \beta_p \partial \beta_q$$

$$= 2 \{ \sum_{i} \sum_{m} (D_{im})'_p (D_{im})'_q D_{im}^{-2} - (D_{im})'_{pq} D_{im}^{-1} \} / g \; ,$$

where

The elements of H are as the following :

For $p = 1,2,3$ and $q = 1,2,3$ and also for $p = 4,5,6$ and $q = 4,5,6$;

$$\partial^2 f_1 / \partial \beta_p \, \partial \beta_q$$

$$= -4(a_{kk}+a_{ko})^{-2} \{ \sum_i \sum_m \sum_j p_{ki}^{-1} d_{km}^{-1} e_{kimj} (e_{kimj})'_p$$

$$+ \sum_{m_k} \sum_j d_{kom_k} e_{kom_k j} (e_{kom_k j})'_p$$

$$\cdot \sum_i \sum_m \sum_j p_{kj}^{-1} d_{km}^{-1} e_{kimj} (e_{kimj})'_q + \sum_{m_k} \sum_j d_{kom_k}^{-1} e_{kom_k j} (e_{kom_k j})'_q \}$$

$$+ 2(a_{kk} + a_{ko})^{-1} \{ \sum_i \sum_m \sum_j p_{ki}^{-1} d_{kmj}^{-1} (e_{kimj})'_p (e_{kimj})'_q$$

$$(D_{im})'_{pq} = \partial D_{im} / \partial \beta_p \, \partial \beta_q$$

$$= (b_{kkim})'_p \, (b_{nnim})'_q \quad , \text{ when } p = 1,2 \text{ and } q = 4,5$$

$$= -(b_{knim})'_p \, (b_{knim})'_q \quad , \text{ wnen } p = 3, \; q = 6$$

$$= \quad 0 \quad , \text{ otherwise.}$$

Rest of the elements are obtained by the property of symmetry.

APPENDIX 4.2

VARIANCES - COVARIANCES OF THE ESTIMATORS

The expressions for the asymptotic variance - covariance matrix for the estimators of the parameters are derived in the following :

The log - likelihood of the observations from IC and MC is

$$\mathcal{L} = \log L$$

$$= \text{constant} + K \sum_i \sum_m \log D_{im} + \sum_k \sum_{m_k} \log b_{kom_k}$$

$$- \frac{1}{2} K(s_1+st) \log V_{10} - \frac{1}{2} K(s_2+st) \log V_{20}$$

$$= \frac{1}{2} \sum_i \sum_m \sum_j (G_{im} \underline{Y}_{imj} - \underline{\varphi}_{oj})' \, V_{oim}^{-1} (G_{im} \underline{Y}_{imj} - \underline{\varphi}_{oj})$$

$$- \frac{1}{2} \sum_k \sum_{m_k} \sum_j V_{ko}^{-1} \{b_{kom_k} Y_{kom_k j} - a_{km_k} \varphi_{koj}\}^2 , \qquad \text{(A4.2.1)}$$

$$i = 1,2,\ldots,t \; ; \; m = 1,2,\ldots, s \; ; \; j = 1,2,\ldots, k \; ;$$

$$m_k = 1,2,\ldots, s_k \; ; \; k = 1,2 \; ;$$

where $\underline{\varphi}_{oj} = (\varphi_{10j}, \varphi_{20j})'$; $\underline{Y}_{imj}$, D_{im} , G_{im} , V_{oim} , b_{kom_k} are as in section 4.3.

In the case of IC

$$E(\underline{Y}_{imj}) = G_{im}^{-1} \underline{\varphi}_{oj} = (I-H_{im})^{-1} P_i D_m \underline{\varphi}_{oj}$$

$$= \underline{a}_{imj} = (a_{1imj} , a_{2imj})', \quad \text{say} , \qquad \text{(A4.2.2)}$$

$$E(Y^2_{kom_kj}) = a^2_{kom_kj} + v_{kom_k} = f_{kom_kj}\text{, say .} \tag{A4.2.7}$$

$$\mathrm{Var}\begin{bmatrix}\hat{\underline{\tau}}\\ \hat{\underline{\nu}}\\ \hat{\underline{\beta}}\end{bmatrix} = \begin{bmatrix} -E\left\langle \begin{matrix} M & O & T \\ O & W & Z \\ T' & Z' & B \end{matrix} \right\rangle \end{bmatrix}^{-1}$$

where

$$\underset{(2k\times 1)}{\hat{\underline{\tau}}} = (\hat{\varphi}_{10}, \hat{\alpha}'_{11}, \ldots, \hat{\alpha}'_{1,k-1}, \hat{\varphi}_{20}, \hat{\alpha}'_{21}, \ldots, \hat{\alpha}'_{2,k-1})',$$

$$\hat{\alpha}'_{n,k-1} = (\hat{\alpha}_{n,k-1} - \hat{\bar{\alpha}}_n) \; ; \; \hat{\varphi}_{ko} = \sum_j \hat{\varphi}_{koj}/K \; ;$$

$$\hat{\underline{\nu}} = (\hat{V}_{10}, \hat{V}_{20})' .$$

$$\underset{(2k\times 2k)}{E(M)} = (E(\partial^2 \pounds / \partial\tau_{1p}, \partial\tau_{1q})),$$

$$= \begin{bmatrix} M_1 & O \\ O & M_2 \end{bmatrix}$$

$$V(\underline{Y}_{imj}) = G_{im}^{-1} V_o (G'_{im})^{-1} = E_{im} \text{, say ,} \qquad \text{(A4.2.3)}$$

and

$$E(\underline{Y}_{imj}\, \underline{Y}'_{imj}) = \underline{a}_{imj}\, \underline{a}'_{imj} + E_{im} = F_{imj} \text{, say ,} \qquad \text{(A4.2.4)}$$

where

$$\underset{(2x2)}{E_{im}} = (e_{knim}) \text{ , } \underset{(2x2)}{F_{imj}} = (f_{knimj}) \text{, } \quad k = 1,2 \text{ ; } \quad n = 1,2 \text{ .}$$

In the case of MC

$$E(Y_{kom_k}) = d_{kom_k} \varphi_{koj}\, b_{kom_k}^{-1} = a_{kom_kj} \text{ , say ,} \qquad \text{(A4.2.5)}$$

$$V(Y_{kom_k}) = d_{kom_k} b_{kom_k}^{-2} V_{ko} = v_{kom_k} \text{ , say ,} \qquad \text{(A4.2.6)}$$

and

where

$$M_k = h_k V_{ko}^{-1} \begin{bmatrix} K & \underline{0}' \\ \underline{0} & (I+J) \end{bmatrix},$$

$\underline{0}$ is the column vector of zeroes of order $(K-1)$,

I is the identity matrix of order $(K-1)$,

J is the matrix with unities of order $(K-1)$,

$$h_k = \left(\sum_k \sum_m p_k d_{km} + \sum_{m_k} d_{kom_k} \right).$$

$$\underset{(2K\times 6)}{E(T)} = (E(\partial^2 \pounds / \partial \tau_p \partial \beta_q)) = E \begin{bmatrix} T_1 & 0 \\ 0 & T_2 \end{bmatrix},$$

where

$E(T_1(p,q))$

$$= \Big[\sum_i \sum_m \sum_j (b_{11im})'_q a_{1imj} + \sum_{m_1} \sum_j (b_{10m_1j})'_q a_{10m_1j} \Big] / V_{10}$$

when $p = 1$ and $q = 1,2,3$.

$$= \Big[\sum_i \sum_m \sum_j (b_{11im})'_q (a_{1im,q-1} - a_{1im_k})$$

$$+ \sum_j (b_{10m_1})'_1 (a_{10m_1,q-1} - a_{10m_1k}) \Big] / V_{10}$$

when $p = 2, \ldots, K$; $q = 1,2,3$.

$E(T_2(p,q))$

$$= \Big[\sum_i \sum_m \sum_j (b_{22im})'_q a_{2imj} + \sum_{m_2} \sum_j (b_{20m_2j})'_q a_{20m_2j} \Big] / V_{20}$$

$E(z(1,1))$

$$= -2 \Big[\sum_i \sum_m \sum_j p_{1i}^{-1} d_{1m}^{c_1 - 1} \{ b_{11im} f_{11imj} + b_{12im} f_{12imj}$$

$$- p_{1i} d_{1m} \varphi_{10j} a_{1imj} \} + \sum_{m_1} \sum_j d_{1m_1}^{c_1 - 1} \{ b_{10m_1} f_{10m_1 j}$$

$$- d_{1m_1} \varphi_{10j} a_{10m_1 j} \} \Big] / V_{10}^2 .$$

$E(z(1,2))$

$$= -2 \Big[\sum_i \sum_m \sum_j (r_{1i} - 1) r_{1i}^{-1} p_{1i}^{-1} d_{1m}^{c_{11} - 1} \{ b_{11im} f_{11imj} + b_{12im} f_{12imj}$$

$$- p_{1i} d_{1m} \varphi_{10j} a_{1imj} \} + \sum_{m_1} \sum \sum_j d_{1m_1}^{c_{11} - 1} \{ b_{10m_1} f_{10m_1}$$

$$- d_{1m_1} \varphi_{10j} a_{10m_1 j} \} \Big] / V_{10}^2 .$$

$$= \Big[\sum_i \sum_m (b_{22im})'_q (a_{2im,q-1} - a_{2imk})$$

$$+ \sum_j \{ (b_{20m_2})'_2 (a_{20m_2,q-1} - a_{20m_2k}) \} \Big] / V_{20}$$

when $p = 2, \ldots, K$ and $q = 4,5,6$.

$$E(W) = (E(W(p,q))) = (E(\partial^2 £ / \partial V_{10} \, \partial V_{20}))$$

$$= -\frac{1}{2} \begin{bmatrix} K\, V_{10}^{-2}(s_1+st) & 0 \\ 0 & K\, V_{20}^{-2}(s_2+st) \end{bmatrix},$$

$$E(Z) = (E(Z(k,p))) = ((E\, \partial^2 £ / \partial V_k \, \partial \beta_p)), \quad k = 1,2,\ldots; \; p = 1,2,3,\ldots,6 \;;$$

where

$$Z = \begin{bmatrix} Z(1,1) & Z(1,2) & Z(1,3) & 0 & 0 & 0 \\ 0 & 0 & 0 & Z(2,4) & Z(2,5) & Z(2,6) \end{bmatrix},$$

$$E(B_1(1,1)) = -K\Big[\sum_i \sum_m \{(D_{im})'_1 / D_{im}\}^2 + \sum_{m_1} \{(b_{10m_1})'_p / b_{10m_1}\}^2\Big]$$

$$-4\Big[\sum_i \sum_m \sum_j p_{1i}^{-1} d_{1m}^{(2c_1-1)} a_{11imj} + \sum_{m_i} \sum_j d_{1m_1}^{(2c_1-1)} a_{10m_kj}\Big]/V_{10},$$

$$E(B_1(2,2)) = -K\Big[\sum_i \sum_m \{(D_{im})'_2 / D_{im}\}^2 + \sum_{m_1} \{(b_{10m_1})'_2 / b_{10m_1}\}^2\Big]$$

$$-4\Big[\sum_i \sum_m \sum_j (r_{1i}-1)^2 r_{1i}^{-2} p_{1i}^{-1} d_{1m}^{(2c_{11}-1)} a_{11imj}$$

$$+ \sum_{m_1} \sum_j d_{1m_1}^{(2c_{11}-1)} a_{10m_1j}\Big] / V_{10},$$

$$E(B_1(3,3)) = -K \sum_i \sum_m \{(D_{im})'_3 / D_{im}\}^2$$

$$-4\Big[\sum_i \sum_m \sum_j r_{2i}^{-2} p_{1i}^{-1} d_{1m}^{-1} d_{2m}^{2c_{12}} a_{12imj}\Big]/V_{10},$$

$$E(z(1,3))$$

$$= -2 \Big[\sum_i \sum_m \sum_j r_{2i}^{-1} p_{1i}^{-1} d_{1m}^{-1} d_{2m}^{c_{12}} \{ b_{11im} f_{12imj}$$

$$+ b_{12im} f_{22imj} - p_{1i} d_{1m} \varphi_{10j} a_{2imj} \} \Big] / V_{10}^2 \, .$$

To get the elements z(2,4), z(2,5) and z(2,6), the subscripts 1 and 2 are to be replaced by 2 and 1, respectively, in the above expressions.

$$E(B) = E(B(p,q)) = E(\partial^2 \pounds / \partial \beta_p \partial \beta_q)$$

$$= E \begin{bmatrix} B_1 & B_2 \\ B_3 & B_4 \end{bmatrix} \quad ; \; p,q = 1,2,\ldots,6 \, .$$

$$E(B_1(2,3)) = -K \left[\sum_i \sum_m (D_{im})'_2 \, (D_{im})'_3 \, D_{im}^{-2} \right]$$

$$-4 \left[\sum_i \sum_m \sum_j p_{1i}^{-1} (r_{1i}-1)(r_{1i} \cdot r_{2i})^{-1} \, d_{1m}^{(c_{11}-1)} \, d_{2m}^{c_{12}} \, a_{12imj} \right] / V_{10}.$$

The expressions for the elements in $E(B_3)$ are same as in $E(B_1)$ except the subscripts 1 and 2 have to be replaced by 2 and 1, respectively.

For $p = 1,2,3$ and $q = 4,5,6$

$$E(B(p,q)) = -K\left[\sum_i \sum_m (D_{im})'_p \, (D_{im})'_q \, D_{im}^{-2} - (D_{im})'_{pq} \, D_{im}^{-1} \right]$$

where $(D_{im})'_p$ and $(D_{im})'_{pq}$ are same as in appendix 4.1.

$$E(B_1(1,2)) = -K \Big[\sum_i \sum_m (D_{im})'_1 \, (D_{im})'_2 \, D_{im}^{-2}$$

$$+ \sum_{m_1} (b_{10m_1})'_1 \, (b_{10m_1})'_2 \, b_{10m_1}^{-2} \Big]$$

$$-4 \Big[\sum_i \sum_m \sum_j p_{1i}^{-1} (r_{1i}-1) r_{1i}^{-1} \, d_{1m}^{(c_1+c_{11}-1)} \, a_{11imj}$$

$$+ \sum_{m_1} \sum_j d_{1m_1}^{(c_1+c_{11}-1)} \, a_{1m_1j} \Big] / V_{10} \, ,$$

$$E(B_1(1,3)) = -K \Big[\sum_i \sum_m (D_{im})'_1 \, (D_{im})'_3 \, D_{im}^{-2} \Big]$$

$$-4 \Big[\sum_i \sum_m \sum_j r_{2i}^{-1} \, p_{1i}^{-1} \, d_{1m}^{(c_1-1)} \, d_{2m}^{c_{12}} \, a_{12imj} \Big] / V_{10} \, ,$$

value to each crop. One can think of an index such as Monetary Equivalent Ratio (MER), which takes into account the monetary value of IC and also of the corresponding MC. LER and MER can be defined in terms of expected yields and denoted by LER_e and MER_e, as

$$LER_e = \sum_k L_{ek} = \sum_k \{E(Y_k)/E(Y_{ko})\} , \qquad (5.1.1a)$$

$$MER_e = \{\sum_k R_k E(Y_k)\} / \{\sum_k R_k p_k E(Y_{ko})\} , \qquad (5.1.1b)$$

where R_k is the cost per unit Y_k, and $p_k = r_k(r_1+r_2)^{-1}$. LER_e and MER_e will be approximately equal to E(LER) and E(MER). MER_e not only depends on the expected yields but also on the price ratio R_1/R_2. When $R_1/R_2 = E(Y_{20})/E(Y_{10})$, MER_e is identical to LER_e. MER_e represents the monetary advantage in IC compared to the situation in which the component crops are grown in MC in the same proportion as in IC. Instead of MER_e one can also use the variate total monetary value (TMV), given by

$$TMV = \sum_k \{R_k E(Y_k)\}.$$

Calculation of LER_{ei} and MER_{ei} for the IC with $r_{1i} : r_{2i}$ row arrangement based on the model (2.2.8) is straight forward. When the effect of plant density is also incorporated the problem of selecting optimum MC yields in calculating these indices also arises. For the kth MC we have from (4.2.2.5) for $c_{kk} = c_k$

$$E(Y_{ko}) = d_{ko} \; \varphi_{ko}(1 - \beta_k' \, d_{ko}^{c_k})^{-1}, \quad (5.1.2)$$

where $\beta'_k = 2(\beta_{ko} + \beta_{kko})$, c_k determines the family of the curve.

$$\lim_{d_{ko}\to 0} E(Y_{ko}) = 0 \text{ for every } c_k ,$$

$$\lim_{d_{ko}\to \infty} E(Y_{ko}) = -\varphi_{ko}\,\beta_k'^{-1} \text{ when } c_k = 1 ,$$

$$= 0 \qquad \text{when } c_k > 1 .$$

Hence the curve given by (5.1.2) is asymptotic when $c_k = 1$ and $E(Y_{ko})$ attains maximum yield given by $(-\varphi_{ko}\,\beta_k'^{-1})$. Whereas in case of $c_k > 1$, $E(Y_{ko})$ attains maximum at a finite d_{ko}, say d^*_{ko}, given by $d^*_{ko} = -\{\beta'_k(c_k-1)\}^{-1/c_k}$. When $c_k < 1$, d^*_{ko} will be negative and is not of interest in our case. Hence when $c_k > 1$ the optimum yield of monocrop k is obtained at the density d^*_{ko}. When $c_k = 1$, the optimum yield is given by $(-\varphi_{10}\,\beta_k'^{-1})$. These can be used in calculating the indices LER_e, MER_e etc.

5.2 METHODS AND EXAMPLES OF OPTIMIZATION

5.2.1 Optimum row arrangement

5.2.1.1 Methods of finding optimum row arrangement

For the ith row arrangement $r_{1i} \vdots r_{2i}$ of the component crops in IC, we have from (2.2.8),

$$E(\underline{Y}_i) = (I-H_i)^{-1} P_i \underline{\mu} .$$

For the kth MC $E(Y_{ko}) = \mu_k(1-2\beta_{kk})^{-1}$.

When r_{1i} or $r_{2i} \to 0$ or ∞, the IC system tends to MC. From the above expected yields, the values for L_{eki} are,

$$L_{e1i} = E(Y_{1i}) / E(Y_{10})$$

$$= (1-2\beta_{11}) D_i^{-1}[\{1-2(r_{2i}-1) r_{2i}^{-1} \beta_{22}\} p_{1i} + 2r_{2i}^{-1} p_{2i} \beta_{12} \mu_1^{-1} \mu_2] \quad (5.1.3a)$$

$$L_{e2i} = E(Y_{2i}) / E(Y_{20})$$

$$= (1-2\beta_{22}) D_i^{-1}[\{1-2(r_{1i}-1) r_{1i}^{-1} \beta_{11}\} p_{2i} + 2r_{1i}^{-1} p_{1i} \beta_{21} \mu_1 \mu_2^{-1}], \quad (5.1.3b)$$

where

$$D_i = |I-H_i| = (1-2\beta_{11}+2r_{1i}^{-1} \beta_{11})(1-2\beta_{22}+2r_{2i}^{-1}\beta_{22}) - 2r_{1i}^{-1}r_{2i}^{-1}\beta_{12}\beta_{21} .$$

Let us consider certain cases of practical interest :

i) When $\beta_{11} = \beta'_{21}$ and $\beta_{22} = \beta'_{12}$ where $\beta'_{mk} = \mu_k \mu_m^{-1} \beta_{mk}$, $k \neq m = 1, 2$, we have,

$$L_{e1i} = (1-2\beta_{11}) D_i^{-1} [(1-2\beta_{22}) + 2\beta_{22}(r_{1i}+r_{2i})(r_{1i} \cdot r_{2i})^{-1}] p_{1i} \quad (5.1.4a)$$

$$L_{e2i} = (1-2\beta_{22}) D_i^{-1}([(1-2\beta_{11}) + 2\beta_{11}(r_{1i}+r_{2i})(r_{1i} \cdot r_{2i})^{-1}] p_{2i} , \quad (5.1.4b)$$

where

$$D_i = (1-2\beta_{11})(1-2\beta_{22}) + 2r_{1i}^{-1}\beta_{11}(1-2\beta_{22}) + 2r_{2i}^{-1} \beta_{22}(1-2\beta_{11}) .$$

From this it follows that $LER_{ei} = L_{e1i} + L_{e2i} = 1$ for every i, and thus there is no biological advantage of IC over MC for any row arrangement.

ii) When $\beta_{11} = \beta_{22} = \beta'_{12} = \beta'_{21}$, then $L_{eki} = p_{ki}$ for every i, i.e. the expected yield of any component crop in IC, irrespective of row arrangements, is same as the expected yield to be obtained through MC.

iii) When $\beta_{11} = \beta_{22}$, then

$$\begin{aligned} LER_{ei} = D_i^{-1}\big[&(1-2\beta_{11})\{(1-2\beta_{11} + 2r_{2i}^{-1}\,\beta_{11})\}p_{1i} \\ &+ (1 - 2\beta_{11} + 2r_{1i}^{-1}\,\beta_{11})\; p_{2i} + (r_{1i}+r_{2i})^{-1}. \\ &\cdot\; (\beta_{12}\,\mu_1^{-1}\mu_2 + \beta_{21}\,\mu_1\,\mu_2^{-1})\;\big] \end{aligned}$$

where

$$\begin{aligned} D_i = &(1 - 2\beta_{11} + 2r_{1i}^{-1}\,\beta_{11})(1 - 2\beta_{11} + 2r_{2i}^{-1}\,\beta_{11}) \\ &- 2(r_{1i}\cdot r_{2i})^{-1}\,\beta_{12}\,\beta_{21}. \end{aligned}$$

From the above relation it follows that LER_e value remains same when r_{1i} and r_{2i} are interchanged for any i.

The coefficients β_{11}, β'_{12}, β_{22} and β'_{21} characterise the degree of intra and interspecific competitions. Apart from the cases mentioned above LER_e depends upon the row arrangements in general. The row arrangements which are of interest in practice are very limited as r_{1i} and r_{2i} can take only integer values. Moreover, as r_{1i} and r_{2i} increase, the component crops in IC tend to behave like MC. One can easily work out LER_{ei}, MER_{ei}, TMV_i or any other $\underline{a}'\,E(\underline{Y}_i)$ for any i. Comparison of these values will help in arriving at the optimum strategy about the row arrangements. For any fixed proportion p_{ki}, there are many possible row arrangements. For

example, when $p_{1i} = p_{2i} = 1/2$, the N possible row arrangements are given by $r_{1i} = r_{2i} = 1, 2, \ldots, N$. Instead of considering variation in r_{1i} and r_{2i} a better insight could be had by studying the effects due to the variation in geometry and proportion p_k. We shall denote the effect of geometry by g, where g is the highest common integer among r_{1i}, r_{2i} such that $(r_{1i} : r_{2i}) = g_i(r_1, r_2)$. For example when $p_{1i} = 1/2$, for $r_{1i} : r_{2i} = 1 : 1$, $g_1 = 1$, for $r_{1i} : r_{2i} = 2 : 2$, $g_i = 2$; when $p_{1i} = 2/3$, for $r_{1i} : r_{2i} = 2 : 3$, $g_i = 1$, for $r_{1i} : r_{2i} = 4 : 6$, $g_i = 2$.

As L_{eki} is the proportion of the expected kth crop yield in IC for the ith row arrangement to its expected MC yield, $(L_{eki} - p_{ki})$ indicates the degree of advantage for crop k. $L_{e1i} \geq p_{1i}$ leads to the condition

$$(\beta'_{12} - \beta_{11}) \geq [\beta_{11}\beta'_{12} - \beta_{11}\beta_{22} + r_{21}^{-1} (\beta_{11}\beta_{22} - \beta_{12}\beta_{21})].$$

A similar relationship holds for L_{e2i}.

5.2.1.2 Some examples

Example 1. Let the values of the parameters be $\beta_{11} = \beta_{22} = -0.40$, $\beta_{12} = \beta_{21} = -0.10$, $\mu_1 = 60$, $\mu_2 = 50$.

The values of LER_e and MER_e worked out from (5.1.1a,b) for different row arrangements are given in Table 5.1a. In this case interspecific competition is sufficiently weak in comparison to intraspecific competition for both the crops. LER_e is maximum for 1 : 1 arrangement as is evident from Table 5.1a. It remains same even if we interchange r_1 and r_2

Table 5.1b Values of LER_e and MER_e (in parenthesis) ... $r_1 : r_2$ at the parameter values $\beta_{11} = -0.40$, $\beta_{12} = -0.10$, $\beta_{22} = -0.6$ $\beta_{21} = -0.40$, $\mu_1 = 60.00$, $\mu_2 = 50.00$.

r_1 \ r_2	1	2	3	4	6	8	10
1	0.945 (0.936)	1.068 (1.063)	1.063 (1.060)	1.055 (1.052)	1.042 (1.040)	1.033 (1.032)	1.028 (1.025)
2	1.078 (1.075)	1.081* (1.079)*	1.068 (1.066)	1.057 (1.056)	1.044 (1.043)	1.035 (1.034)	1.030 (1.028)
3	1.075 (1.073)	1.069 (1.068)	1.059 (1.058)	1.051 (1.050)	1.040 (1.040)	1.033 (1.032)	1.028 (1.027)
4	1.065 (1.064)	1.060 (1.059)	1.052 (1.051)	1.046 (1.045)	1.037 (1.036)	1.031 (1.030)	1.026 (1.026)
6	1.050 (1.050)	1.046 (1.046)	1.041 (1.041)	1.037 (1.037)	1.031 (1.031)	1.027 (1.026)	1.023 (1.023)
8	1.040 (1.040)	1.037 (1.037)	1.034 (1.034)	1.031 (1.031)	1.027 (1.027)	1.024 (1.023)	1.021 (1.021)
10	1.034 (1.033)	1.031 (1.031)	1.029 (1.029)	1.027 (1.027)	1.024 (1.023)	1.021 (1.021)	1.019 (1.019)

Note. * indicates the maximum values of LER_e and MER_e.

Table 5.1b Values of LER_e and MER_e (in parenthesis) for different row arrangements $r_1 : r_2$ at the parameter values $\beta_{11} = -0.40$, $\beta_{12} = -0.10$, $\beta_{22} = -0.60$, $\beta_{21} = -0.40$, $\mu_1 = 60.00$, $\mu_2 = 50.00$.

r_1 \ r_2	1	2	3	4	6	8	10
1	0.945 (0.936)	1.068 (1.063)	1.063 (1.060)	1.055 (1.052)	1.042 (1.040)	1.033 (1.032)	1.028 (1.025)
2	1.078 (1.075)	1.081* (1.079)*	1.068 (1.066)	1.057 (1.056)	1.044 (1.043)	1.035 (1.034)	1.030 (1.028)
3	1.075 (1.073)	1.069 (1.068)	1.059 (1.058)	1.051 (1.050)	1.040 (1.040)	1.033 (1.032)	1.028 (1.027)
4	1.065 (1.064)	1.060 (1.059)	1.052 (1.051)	1.046 (1.045)	1.037 (1.036)	1.031 (1.030)	1.026 (1.026)
6	1.050 (1.050)	1.046 (1.046)	1.041 (1.041)	1.037 (1.037)	1.031 (1.031)	1.027 (1.026)	1.023 (1.023)
8	1.040 (1.040)	1.037 (1.037)	1.034 (1.034)	1.031 (1.031)	1.027 (1.027)	1.024 (1.023)	1.021 (1.021)
10	1.034 (1.033)	1.031 (1.031)	1.029 (1.029)	1.027 (1.027)	1.024 (1.023)	1.021 (1.021)	1.019 (1.019)

Note. * indicates the maximum values of LER_e and MER_e.

because $\beta_{11} = \beta_{22}$. However, partial LER_e values L_{1e} or L_{2e} are different when r_1 and r_2 are interchanged. The parameters β_{11}, β'_{12}, β_{22} and β'_{21} characterise the degree of competition. For this case $\beta'_{12} = -0.083$ and $\beta'_{21} = -0.120$. Though $\beta_{11} = \beta_{22}$ and $\beta_{12} = \beta_{21}$, LER_e values depend upon r_1 and r_2 since $\beta'_{12} \neq \beta'_{21}$. In Table 5.1a MER_e has been calculated for $R_1 = 2$ and $R_1 = 3$. In this case MER_e is also maximum for 1 : 1 arrangement. Reason for such a conclusion is not difficult to see from the values of the coefficients β_{11}, β_{22}, β'_{12} and β'_{21} which suggest that the competition pressure per unit area is less when the neighbouring rows belong to the other component crop than of its own type.

Example 2. Let the values of the parameters be $\beta_{11} = -0.40$, $\beta_{22} = -0.60$, $\beta_{12} = -0.10$, $\beta_{21} = -0.4$, $\mu_1 = 60$, $\mu_2 = 50$.

For this set both LER_e and MER_e (Table 5.1b) are minimum for 1 : 1 row arrangement and are less than unity i.e. MC is advantageous over 1 : 1 row arrangement. In this case 2:2 arrangement is better than the other arrangements. Here $\beta'_{12} = 0.083$ and $\beta'_{21} = 0.480$. The value of β'_{21} is considerably large in comparison to its value in example 1. This is reflected in LER_e and MER_e values for various row arrangements.

From Tables 5.1a,b it is clear that as r_1 and r_2 increase, LER_e and MER_e tend to unity i.e. they tend to the values of MC in both the examples. Figures 5.1 and 5.2 give the values of LER_e and MER_e for different values of geometry (g) of the

component crop rows for the two sets of parameters. g^{-1} indicates the degree of intimacy between two component crop rows and it varies between zero and one. Intimacy is maximum when g^{-1} is one and minimum when it is zero. From figures 5.1 and 5.2 it is evident that LER_e and MER_e tend to unity as g increases. These figures give insight into the dependence of LER_e and MER_e on the intimacy between the two component crops. LER_e and MER_e vary more steeply at the lower values of g but the changes at the higher level of g are small. For the first set of parameters the values of LER_e are symmetrical around $p_1 = 0.5$, consequently figures are drawn only for $p_1 \leq 0.50$.

5.2.2 Optimum row arrangement and plant density

Let us now consider the problem of finding simultaneously the optimum combination of row arrangement and plant density levels for both the component crops which maximize $Z = \underline{a}' E(\underline{Y})$. Z may be LER_e, TMV_e or any other meaningful linear combination of the component crop yields. Since d_1 and d_2 take continuous values and Z is a complicated function in d_1 and d_2, we are required to use some nonlinear optimization methods for finding optimum d_1 and d_2. When $c_1 = c_{11} = c_{21}$ and $c_2 = c_{22} = c_{12}$, for fixed r_{1i} and r_{2i}, we have from (4.6.3),

$$E(\underline{Y}) = (I-H)^{-1} D \underline{\theta},$$

where

$$(I-H)^{-1} = D_e^{-1} \begin{bmatrix} 1-\beta_2 d_2^{c_2} & \beta_{12} d_2^{c_2} \\ \beta_{21} d_1^{c_1} & 1-\beta_1 d_1^{c_1} \end{bmatrix}, \quad \underline{Y}' = (Y_1, Y_2), \quad \underline{\varphi}' = (\varphi_1, \varphi_2)$$

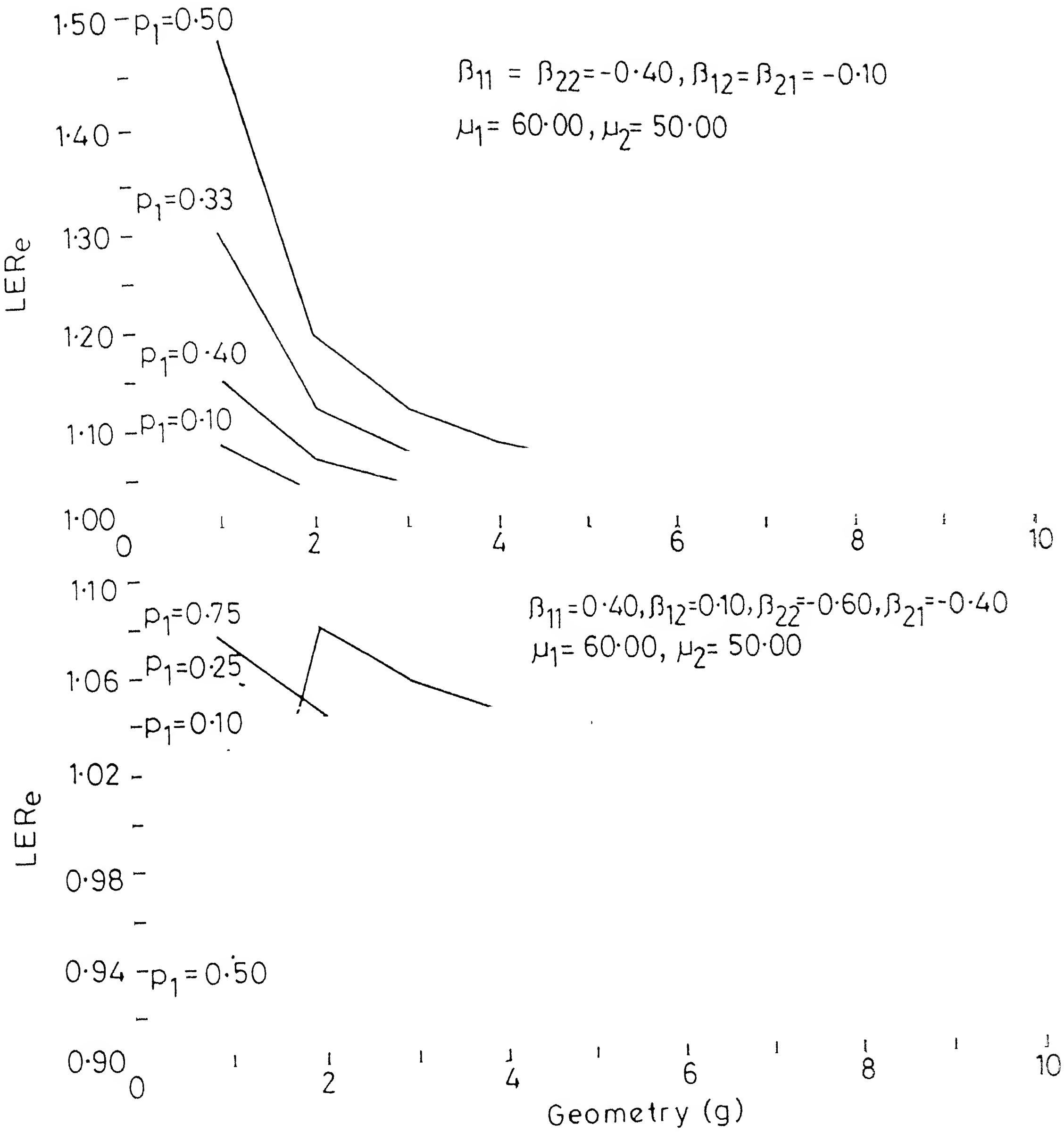


Fig. 5·1 Values of LER_e for different geometry (g) and proportion p_1

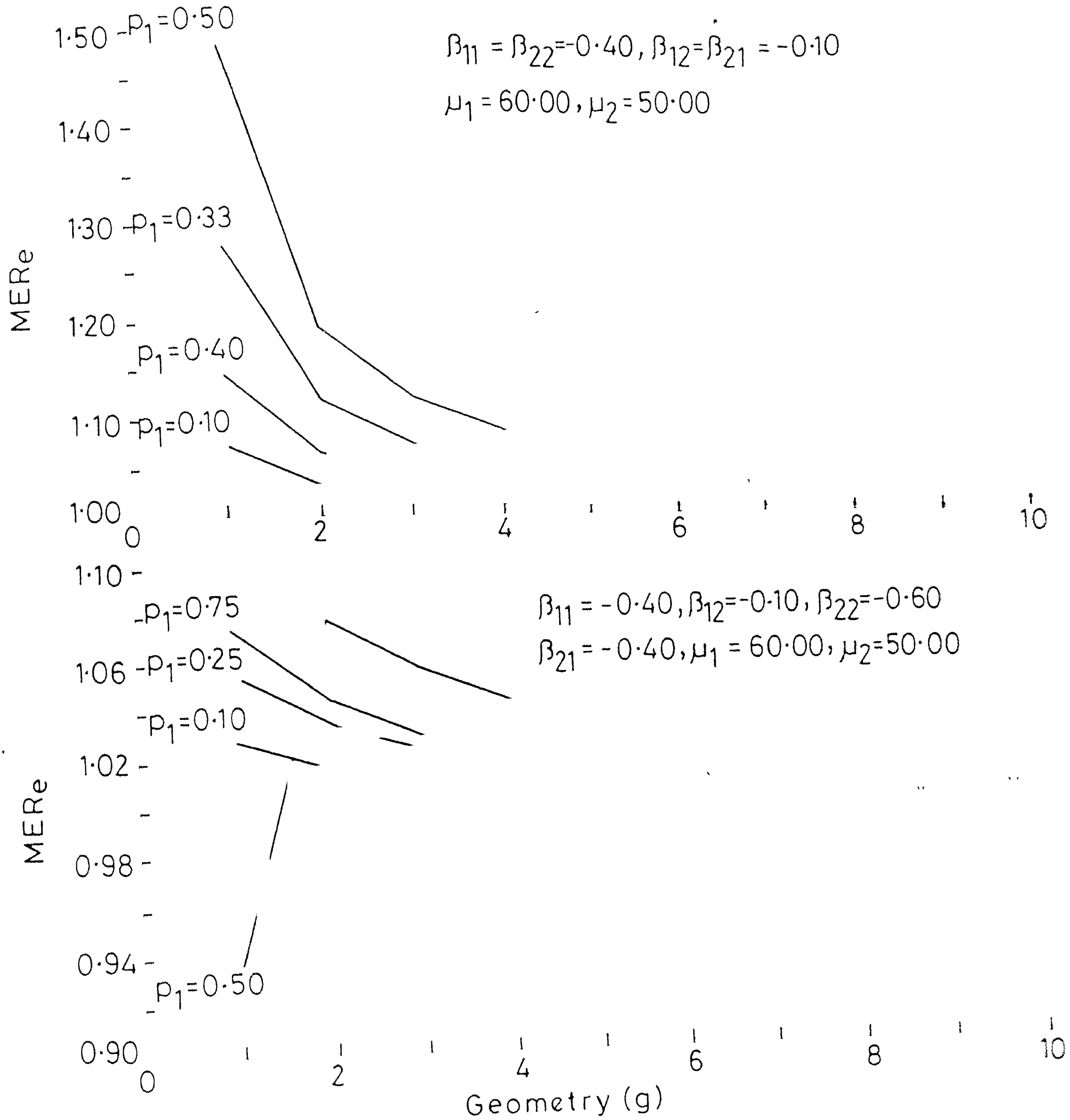


Fig. 5·2 Values of MER_e for different geometry (g) and proportion p_1 at the prices $R_1=2$ and $R_2=3$ of crop 1 and crop 2 respectively

$$\lim_{d_1 \to o} E(Y_2) = d_2\varphi_2(1-\beta_2 d_2^{c_2})^{-1} .$$

ence as $d_2 \to o$, $E(Y_1)$ tends to the expected value of the MC 1. imilarly as $d_1 \to o$, $E(Y_2)$ tends to the expected value of the C 2.

The maximum values of the linear combinations $Z_1 = E(Y_1)+2E(Y_2)$, $Z_2 = E(Y_1) + 7E(Y_2)$ and LER_e, when maximised for d_1 and d_2, at the parameter values $\beta_{10} = \beta_{20} = -0.40$, $\beta_{110} = \beta_{220} = -0.30$, $\beta_{120} = \beta_{210} = -0.01$, $\varphi_{10} = 60$ and $\varphi_{20} = 40$, are given in Table 5.2a, b and c for different r_1, r_2, c_1 and c_2. The maximum over d_1 and d_2 has been obtained by using the simplex algorithm. To confirm the results obtained by this procedure the function values in the neighbourhood of the optimum value

$$D_e = (1-\beta_1 d_1^{c_1})(1-\beta_2 d_2^{c_2}) - \beta_{12}\beta_{21} d_1^{c_1} d_2^{c_2},$$

$$\varphi_k = p_{ki}\,\varphi_{10}, \quad \beta_k = 2\beta_{ko} + 2(r_{ki}-1)\, r_{ki}^{-1}\,\beta_{kko},$$

$$\beta_{mk} = 2r_k^{-1}\,\beta_{mko} \quad \text{for } m \neq k = 1, 2.$$

e individual component crop means are given by

$$E(Y_1) = D_e^{-1}\left[(1-\beta_2 d_2^{c_2})\, d_1\,\varphi_1 + \beta_{12}\, d_2^{c_2+1}\,\varphi_2\right],$$

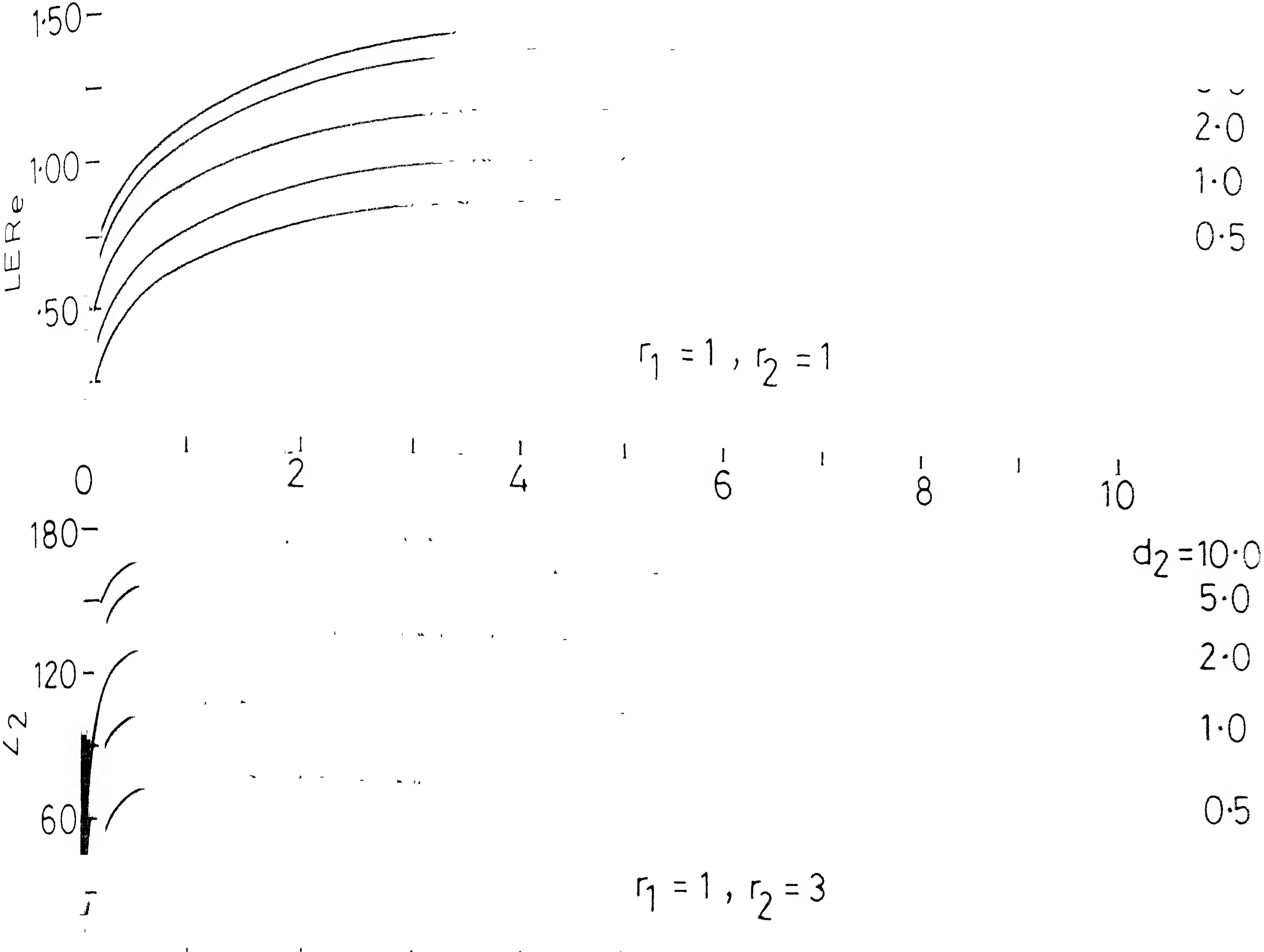
$$E(Y_2) = D_e^{-1}\left[(1-\beta_1 d_1^{c_1})\, d_2\varphi_2 + \beta_{21}\, d_1^{c_1+1}\,\varphi_1\right].$$

rom this it follows that

$$\lim_{d_2 \to 0} E(Y_1) = d_1\varphi_1(1-\beta_1 d_1^{c_1})^{-1},$$

are examined graphically. When $\beta_{12} = \beta_{21} = 0$, $c_1 = c_2 = 1$ and d_1 and d_2 tend infinity, Z approaches asymptotically to the maximum yield $-(a_1 \varphi_1 \beta_1^{-1} + a_2 \varphi_2 \beta_2^{-1})$. It tends to zero when c_1 and $c_2 > 1$. Nature of the function is such that when β_{12} and β_{21} are close to zero the optimum d_1 and d_2 occur at larger values of d_1 and d_2. This is also reflected in Tables 5.2a, b and c. When $c_1 = 1$ optimum d_1 occured at relatively larger value of d_1 compared to the situation when $c_1 > 1$. In practice the parameters have to be estimated from the experimental data. In situations when the optimum lies outside the range of the experiment it is advisable to repeat the experiment by including levels around the predicted optimum d_1 and d_2.

The optimum row arrangement for Z_1, Z_2 and LER_e can be found by comparing their values over different r_1 and r_2 obtained by maximizing over d_1 and d_2. For the linear combination Z_1 and LER_e the optimum row arrangement is 1:1, while for Z_2 1:3 arrangement is optimum. For example, the nature of the curves of Z_1, Z_2 and LER_e for variation in d_1 and d_2 at their optimum row arrangements, are shown in Figures 5.3 ($c_1 = c_2 = 2$) and 5.4 ($c_1 = c_2 = 1$). We see from Figure 5.4 that Z_1, Z_2 and LER_e do not attain their maxima even when $d_1 = 10$ and $d_2 = 10$, whereas in Figure 5.3 maximum value is attained at relatively smaller values of d_1 and d_2.



1·50
1·00
·50
LERe
$r_1 = 1$, $r_2 = 1$
2·0
1·0
0·5
0
2
4
6
8
10
180
120
60
∠2
$d_2 = 10·0$
5·0
2·0
1·0
0·5
$r_1 = 1$, $r_2 = 3$

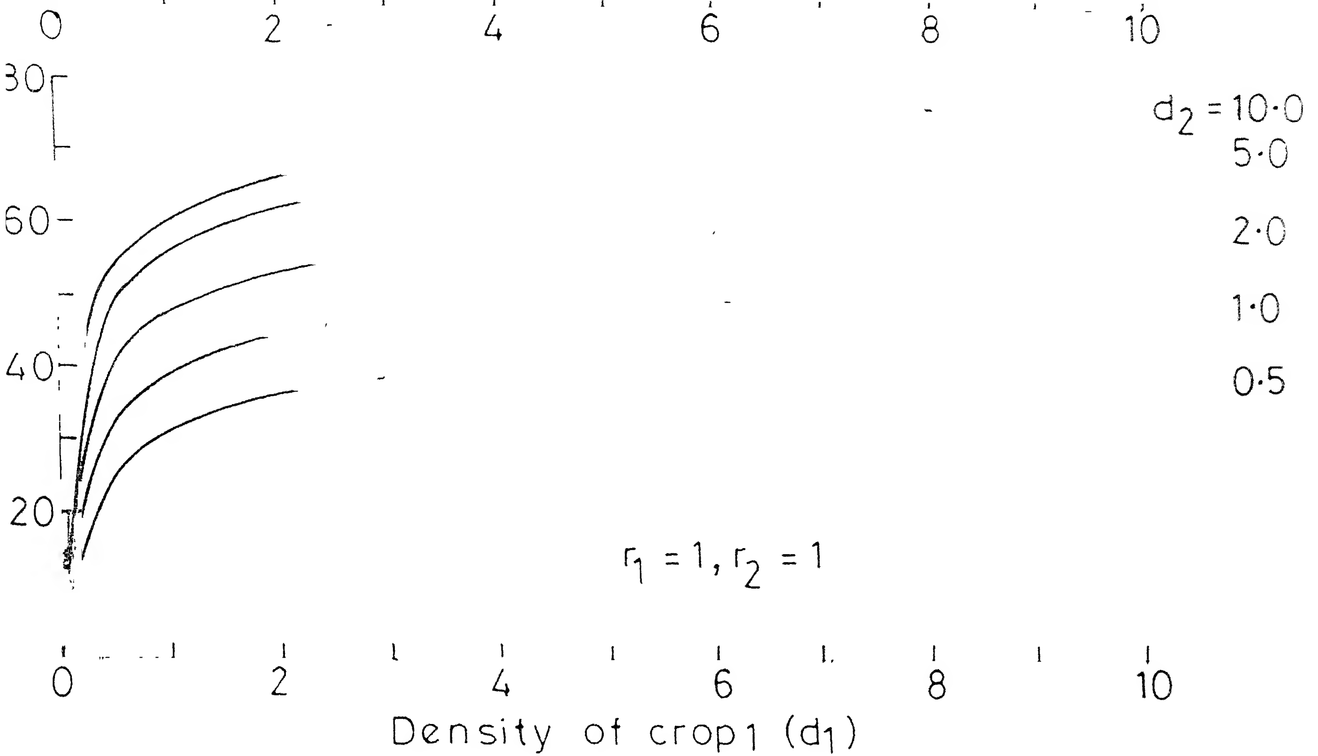


Fig. 5·4 LER_e, $Z_1 = E(y_1) + 2E(y_2)$, $Z_2 = E(y_1) + 7E(y_2)$ for different plant densities d_1 and d_2 at $\beta_{10} = \beta_{20} = -0{\cdot}40$, $\beta_{110} = \beta_{220} = -0{\cdot}30$, $\beta_{120} = \beta_{210} = -0{\cdot}01$, $\phi_{10} = 60$, $\phi_{20} = 40$, $C_1 = C_2 = 1$

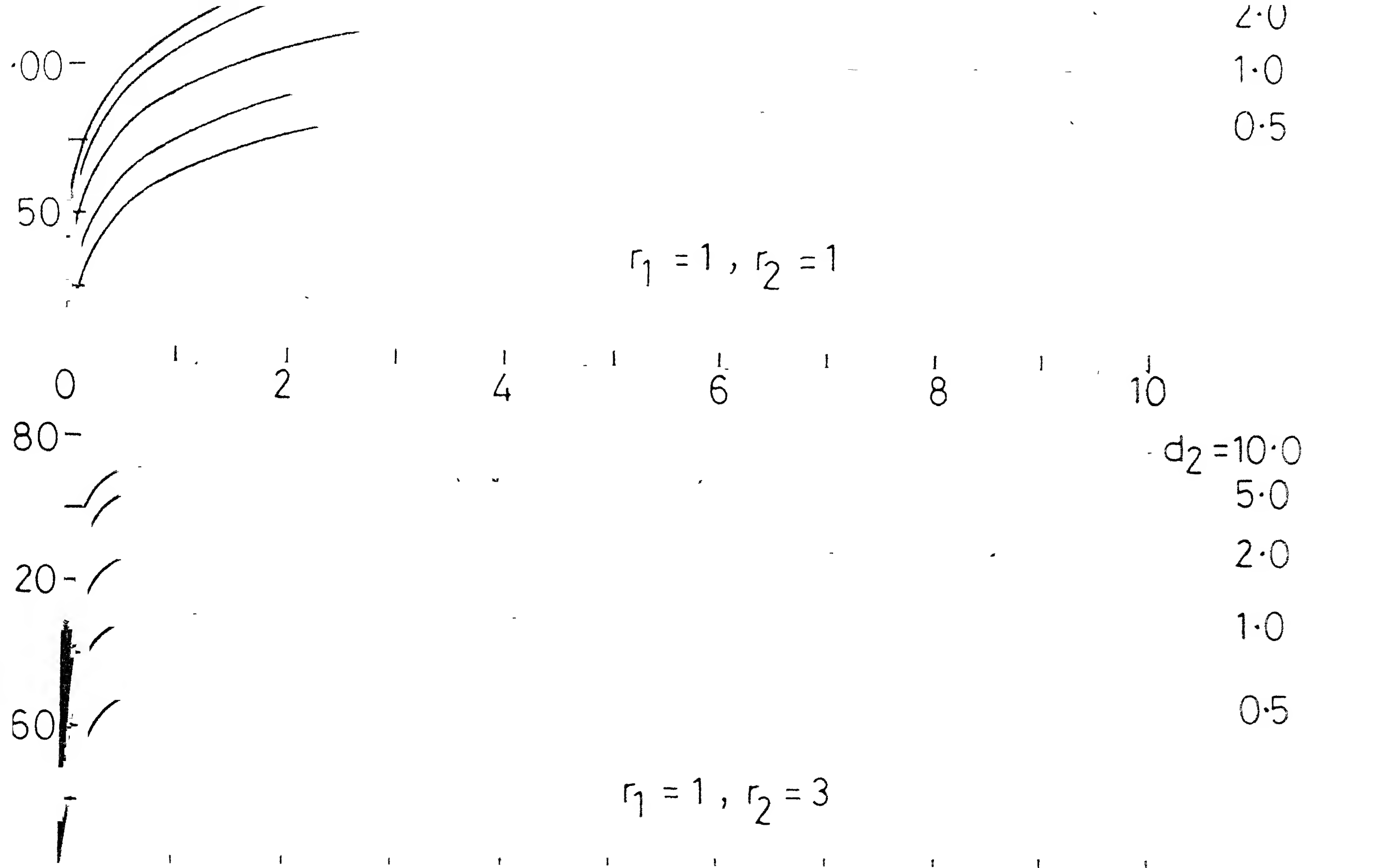

·00
50
0
2
4
6
8
10
2·0
1·0
0·5
$r_1 = 1$, $r_2 = 1$
80
20
60
$d_2 = 10·0$
5·0
2·0
1·0
0·5
$r_1 = 1$, $r_2 = 3$

Table 5.2a Maximum of Z_1, Z_2 and LER_e at their optimum densities d_1^* and d_2^*, for various row arrangements. $\beta_{10} = \beta_{20} = -0.40$, $\beta_{110} = \beta_{220} = -0.30$, $\beta_{120} = \beta_{210} = -0.01$, $\varphi_{10} = 60.00$, $\varphi_{20} = 40.00$, $c_1 = c_2 = 2$.

Row arrangements		Z_1			Z_2			LER_e		
r_1	r_2	d_1^*	d_2^*	Z_1^*	d_1^*	d_2^*	Z_2^*	d_1^*	d_2^*	LER_e^*
1	1	1.08	1.13	38.59**	0.97	1.12	93.63	1.13	1.14	1.30 **
1	2	1.06	0.96	36.28	0.96	0.96	99.27	1.12	0.96	1.18
1	3	1.06	0.92	35.53	0.97	0.92	103.57**	1.07	0.94	1.13
2	1	0.94	1.11	33.71	0.91	1.12	70.58	0.97	1.06	1.18
2	2	0.94	0.95	33.20	0.89	0.96	80.57	0.89	0.99	1.12
2	3	0.94	0.91	33.22	0.91	0.91	87.75	0.95	0.95	1.09
3	1	0.90	0.93	31.56	0.89	1.12	59.22	0.90	1.03	1.13
3	2	0.91	0.96	31.56	0.88	0.95	69.48	0.92	0.92	1.09
3	3	0.92	0.93	31.84	0.88	0.91	77.31	0.93	0.88	1.08
4	4	0.89	0.89	31.23	0.85	0.89	75.82	0.88	0.92	1.06

Note. ** indicates overall optimum values

$$Z_1 = E(Y_1) + 2E(Y_2),\quad Z_2 = E(Y_1) + 7E(Y_2)$$

$$Z_1^* = Z_1(d_1^*, d_2^*),\quad Z_2^* = Z_2(d_1^*, d_2^*),\quad LER_e^* = LER_e(d_1^*, d_2^*)$$

Table 5.2c Maximum of Z_1, Z_2 and LER_c at their optimum densities d_1^* and d_2^* for various row arrangements. $\beta_{10} = \beta_{20} = -0.40$, $\beta_{110} = \beta_{220} = -0.30$, $\beta_{120} = \beta_{210} = -0.01$, $\varphi_{10} = 60.00$, $\varphi_{20} = 40.00$, $c_1 = 1$ and $c_2 = 2$.

Row arrangements		Z_1			Z_2			LER_e		
r_1	r_2	d_1^*	d_2^*	Z_1^*	d_1^*	d_2^*	Z_2^*	d_1^*	d_2^*	LER_e^*
1	1	7.86	1.38**	50.16	3.36	1.21	99.35	6.15	1.39	1.28**
1	2	7.34	1.08	43.84	3.23	1.01	103.00	6.18	1.00	1.16
1	3	6.18	0.96	41.11	3.18	0.94	106.34**	6.14	0.95	1.12
2	1	9.91	1.38	45.26	4.59	1.22	77.85	8.95	1.40	1.13
2	2	9.20	1.08	41.73	4.40	1.00	85.94	9.02	1.03	1.08
2	3	9.13	0.99	40.01	4.33	0.95	92.02	6.51	0.83	1.05
3	1	12.67	1.47	43.77	5.55	1.24	67.72	10.61	1.51	1.08
3	2	11.11	1.08	41.23	5.36	1.01	76.20	10.29	1.00	1.04
3	3	10.72	1.00	39.87	5.31	0.95	82.88	10.30	1.00	1.03
4	4	12.25	0.96	39.09	6.05	0.92	81.60	10.33	1.00	1.02

Note. ** indicates overall optimum values

$Z_1 = E(Y_1) + 2E(Y_2)$, $Z_2 = E(Y_1) + 7E(Y_2)$

$Z_1^* = Z_1(d_1^*, d_2^*)$, $Z_2^* = Z_2(d_1^*, d_2^*)$, $LER_e^* = LER_e(d_1^*, d_2^*)$

5.3 DISCUSSION

In this chapter some aspects of optimizing certain functions of component crop yields such as LER_e, MER_e or any other linear combination of the component crop yields have been discussed. In practice it may be desirable to impose certain constraints which component crop yields must satisfy. In such situation modified indices of LER suggested by Mead and Stern (1979), Reddy and Chetty (1984) may be more appropriate. These modified indices can be worked out in terms of the expected yields as in the case of LER_e and MER_e.

Use of nonlinear optimization methods in finding the optimum plant density levels require some care. One may have to verify that the maximum thus obtained is a local maximum or a global maximum. The functions considered here are generally concave functions of d_1 and d_2 and there exists a unique maximum. This has also been verified by plotting the values over different d_1 and d_2. In practice the cost of inputs are also to be considered hence, optimization subject to these economic constraints may be more appropriate. The numerical results along with the graphical representations will help in understanding the response of component crop yields to various row arrangements and density levels. This will subsequently help in arriving at their optimum values.

In practice the parameters have to be estimated through experimental data. The precision of the estimates depends on the amount of information available in the data. Large sample variance-covariance matrix and simulation results discussed in the third and fourth chapters give some idea on these aspects. Extrapolation should be avoided as the model discussed here may not hold good for very high values of plant density and row arrangements.

CHAPTER VI

ANALYSIS OF EXPERIMENTAL DATA

6.1 INTRODUCTION

Eventhough the model developed in the second and fourth chapters are based on the principles of plant competition the goodness of fit of these models to the experimental data collected under field conditions need to be examined. As the models involve a large number of parameters, it is of considerable importance to examine whether the model can be fitted adequately by smaller number of parameters. For example, when the component crops behave independently in IC, the usefulness of interspecific competition coefficients has to be tested. Likelihood ratio criterion, and test based on asymptotic standard errors, can be used for testing whether the competition parameters are different from zero or not.

Alternatively, Akaike (1974) information criterion (AIC), defined by

$$AIC = -2 \log L + 2(\text{number of parameters}) ,$$

appears to be a useful technique in judging the adequate number of parameters to be fitted in the model. This criterion adjusts the log-likelihood for the number of parameters to be estimated. AIC is derived under the assumption that the true distribution can be described by the given model when

Table 5.2c Maximum of Z_1, Z_2 and LER_c at their optimum densities d_1^* and d_2^* for various ro arrangements. $\beta_{10} = \beta_{20} = -0.40$, $\beta_{110} = \beta_{220} = -0.30$, $\beta_{120} = \beta_{210} = -0.01$, $\varphi_{10} = 60.00$, $\varphi_{20} = 40.00$, $c_1 = 1$ and $c_2 = 2$.

Row arrangements		Z_1			Z_2			LER_e		
r_1	r_2	d_1^*	d_2^*	Z_1^*	d_1^*	d_2^*	Z_2^*	d_1^*	d_2^*	LER_e^*
1	1	7.86	1.38**	50.16	3.36	1.21	99.35	6.15	1.39	1.28**
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1	3	6.18	0.96	41.11	3.18	0.94	106.34**	6.14	0.95	1.12
2	1	9.91	1.38	45.26	4.59	1.22	77.85	8.95	1.40	1.13
2	2	9.20	1.08	41.73	4.40	1.00	85.94	9.02	1.03	1.08
2	3	9.13	0.99	40.01	4.33	0.95	92.02	6.51	0.83	1.05
3	1	12.67	1.47	43.77	5.55	1.24	67.72	10.61	1.51	1.08
3	2	11.11	1.08	41.23	5.36	1.01	76.20	10.29	1.00	1.04
3	3	10.72	1.00	39.87	5.31	0.95	82.88	10.30	1.00	1.03
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$Z_1 = E(Y_1) + 2E(Y_2)$, $Z_2 = E(Y_1) + 7E(Y_2)$

$Z_1^* = Z_1(d_1^*, d_2^*)$, $Z_2^* = Z_2(d_1^*, d_2^*)$, $LER_e^* = LER_e(d_1^*, d_2^*)$

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Use of nonlinear optimization methods in finding the optimum plant density levels require some care. One may have to verify that the maximum thus obtained is a local maximum or a global maximum. The functions considered here are generally concave functions of d_1 and d_2 and there exists a unique maximum. This has also been verified by plotting the values over different d_1 and d_2. In practice the cost of inputs are also to be considered hence, optimization subject to these economic constraints may be more appropriate. The numerical results along with the graphical representations will help in understanding the response of component crop yields to various row arrangements and density levels. This will subsequently help in arriving at their optimum values.

In practice the parameters have to be estimated through experimental data. The precision of the estimates depends on the amount of information available in the data. Large sample variance-covariance matrix and simulation results discussed in the third and fourth chapters give some idea on these aspects. Extrapolation should be avoided as the model discussed here may not hold good for very high values of plant density and row arrangements.

CHAPTER VI

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6.1 INTRODUCTION

Eventhough the model developed in the second and fourth chapters are based on the principles of plant competition the goodness of fit of these models to the experimental data collected under field conditions need to be examined. As the models involve a large number of parameters, it is of considerable importance to examine whether the model can be fitted adequately by smaller number of parameters. For example, when the component crops behave independently in IC, the usefulness of interspecific competition coefficients has to be tested. Likelihood ratio criterion, and test based on asymptotic standard errors, can be used for testing whether the competition parameters are different from zero or not.

Alternatively, Akaike (1974) information criterion (AIC), defined by

$$AIC = -2 \log L + 2(\text{number of parameters}) ,$$

appears to be a useful technique in judging the adequate number of parameters to be fitted in the model. This criterion adjusts the log-likelihood for the number of parameters to be estimated. AIC is derived under the assumption that the true distribution can be described by the given model when

its parameters are suitably adjusted. The model with the minimum AIC gives the best fit. Examination of residuals is another way of looking at the goodness of fit of the model. Unlike the Likelihood ratio or Akaike measure this does not depend upon the form of the alternative model.

In section 6.2 some aspects of fitting the model developed in the fourth chapter which include the factors such as row arrangement and plant densities shall be discussed. The data are from an IC experiment on mustard and chickpea (Kushwaha, 1983). The experiment involves a very limited number of row arrangements and plant density levels. These data have been used for testing the goodness of fit of the model developed in previous chapters, in the absence of data from better designed experiments.

6.2 ANALYSIS

6.2.1 Experimental details

The experiment has sixteen treatments with four replications each. Randomized block design was adopted. Among the sixteen treatments four treatments are of MC on mustard, four treatments are of MC on chickpea and the remaining eight are on IC of the above two. In each MC there are four plant density levels. In IC there are two row arrangements and four density levels for each row arrangement. In MC plots interrow distances are 60 and 30 cms for mustard and chickpea, respectively.

The two row arrangements in IC are $r_{1i} : r_{2i} = 1 : 4, 2 : 2$, where r_{1i} and r_{2i} are the number of rows of mustard and chickpea in the ith row arrangement. In IC plots the distance between a row of chickpea and mustard is 45 cm. The area occupied by one row of mustard is equal to the area occupied by two rows of chickpea. The row arrangements in MC and IC are illustrated in Figure 6.1. The plant density levels are obtained by adjusting the space between the plants within a row. The four plant density levels for mustard and chickpea are :

$$\rho_{1m} = \rho_{2m} = 7.5,\ 15.0,\ 22.5 \text{ and } 30.0 \text{ plants/m}^2 .$$

They have been coded as d_{1m} and d_{2m} given by

$$d_{1m} = d_{2m} = 1,2,3,4 ;$$

where $d_{km} = \rho_{km}/7.5 , \quad k = 1,2.$

The plant density levels for each crop in MC are same as in IC . Plotwise yield data are given in appendix 6.1.

6.2.2 Fitting of the model

The data have been analysed by fitting the models (4.3.1a,b) and (4.3.4a,b) for IC and MC, respectively. The models are as the following :

for IC

$$(I-H_{im})\ \underline{Y}_{imj} = P_i\ D_{im}\ \underline{\varphi}_{oj} + \underline{\varepsilon}_{imj} , \quad i = 1,2 ; \ m,j = 1,2,\dots,4 ;$$

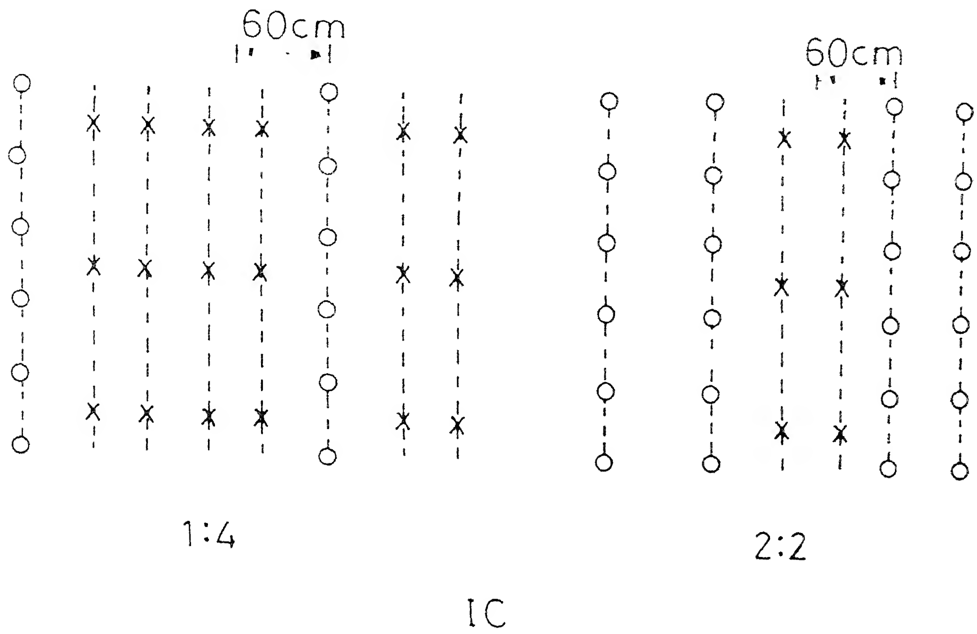
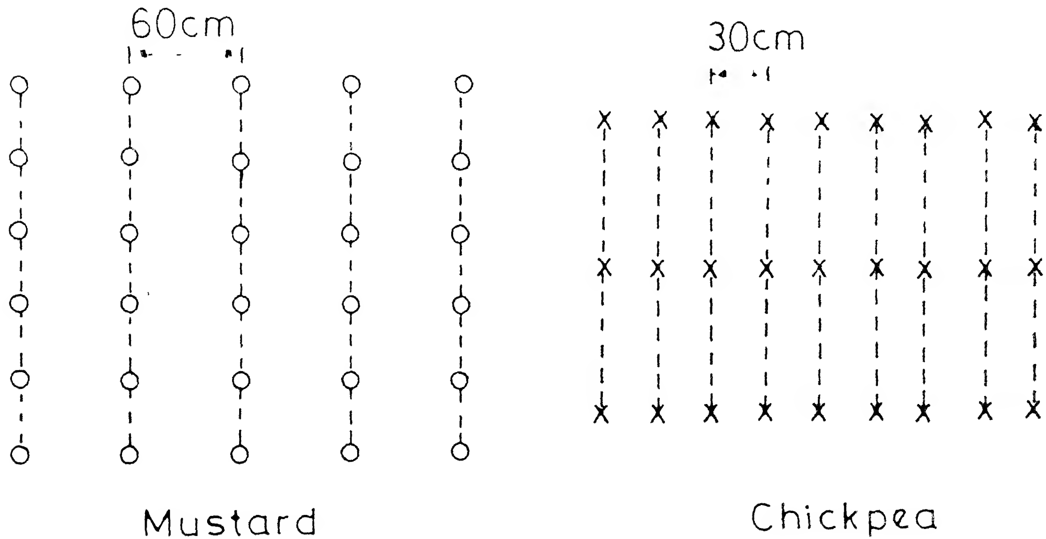


Fig. 6·1 Arrangement of mustard and Chickpea rows in monocropping (MC) and intercropping (IC)



MC

for MC

$$(1 - 2\beta_{ko}\, d_{km_k}^{c_k} - 2\beta_{kko}\, d_{km_k}^{c_{kk}})\, Y_{kom_kj} = d_{km_k}\, \varphi_{koj} + \varepsilon_{kom_kj}$$

$$k = 1,2\ ;\ m_k\ ,\ j = 1,2,3,4\ ;$$

where H_{im} , D_{im} , P_i , $\underline{\varphi}_{oj}$, $\underline{\varepsilon}_{imj}$, $\underline{Y}_{imj}$, d_{km_k} are as mentioned earlier. Here we assume $c_1 = c_{11} = c_{21}$ and $c_2 = c_{22} = c_{12}$. In the present experimental situation $d_{1m} = d_{2m} = d_{1m_1} = d_{2m_2}$ for m , m_1 , $m_2 = 1,2,3,4$;

$$p_{1i} = 2r_{1i}(2r_{1i} + r_{2i})^{-1}\ ,\ p_{2i} = r_{2i}(2r_{1i} + r_{2i})^{-1}\ .$$

The log-likelihood for all the data is given by

$$\begin{aligned}
\log L = -\{ & 48 \log(2\pi) + 2(\sum_m \log d_{1m} + \sum_m \log d_{2m}) \\
& + 2 \sum_i \sum_m \log(p_{1i}\, p_{2i}\, d_{1m}\, d_{2m}) \} \\
& + 4 \sum_i \sum_m \log |I-H_{im}| + 4 \sum_k \sum_m b_{km} - 24 \log(V_{10} \cdot V_{20}) \\
& - \frac{1}{2} \sum_i \sum_m \sum_j [\ \{ (I-H_{im})\, \underline{Y}_{imj} - P_i\, D_m\, \underline{\varphi}_{oj} \}'\ P_i^{-1}\, D_m^{-1}\, V_o^{-1} \\
& \cdot \{ (I-H_{im})\, \underline{Y}_{imj} - P_i\, D_m\, \underline{\varphi}_{oj} \}] \\
& - \frac{1}{2} \sum_k \sum_m \sum_j [\ \{ b_{km}\, Y_{kom} - d_{km}\ \ _{koj} \}^2]\ ,
\end{aligned}$$

where $\quad b_{km} = 1 - 2(\beta_{ko} + \beta_{kko})\, d_{1k}^{c_k}$.

The parameters are estimated by using the iterative procedure discussed in appendix 4.1. The initial values are obtained by simplex procedure. The convergence

criterion used is same as in section 3.5. Iterative procedure was repeated for various values of c_1 and c_2 since there was a problem in the convergence for these data in obtaining the ML estimates of c_1 and c_2. The value of $-2 \log L$ is near its minimum when $c_1 = c_2 = 1.5$.

The estimates of the parameters with their asymptotic standard errors are :

Parameter	β_{10}	β_{110}	β_{120}	β_{20}	β_{220}	β_{210}	φ_{10}	φ_{20}	v_{10}	v_{20}
Estimate	-0.428	-0.178	-0.069	0.073	-0.526	-0.086	3.142	6.870	0.223	0.407
Asymptotic standard error	0.100	0.101	0.023	0.046	0.065	0.278	0.353	0.407	0.054	0.094

Here $\quad \varphi_{k_0} = \sum_{j=1}^{4} \varphi_{k0j}/4$.

The competition parameters β_{10}, β_{120} and β_{220} are significantly different from zeroes.

The estimate of the asymptotic variance-covariance matrix of $\hat{\underline{\beta}}$ coefficients is given in the upper triangle of the following matrix :

$$V(\hat{\underline{\beta}}) = 10^{-3} \begin{bmatrix} 10.09 & -6.83 & -1.19 & -0.31 & 1.03 & 5.21 \\ & 10.22 & 2.03 & 0.97 & -1.22 & -6.20 \\ & & 0.51 & 0.22 & -0.28 & -1.41 \\ & & & 2.10 & -2.65 & -12.03 \\ & & & & 4.28 & 15.34 \\ & & & & & 77.20 \end{bmatrix} .$$

The correlations are very high among $\hat{\beta}_{10}$, $\hat{\beta}_{110}$, $\hat{\beta}_{120}$ and also among $\hat{\beta}_{20}$, $\hat{\beta}_{220}$ and $\hat{\beta}_{210}$, which can be seen from the following correlation matrix Corr.$(\hat{\underline{\beta}})$ (given in the upper triangle)

$$\text{Corr.}(\hat{\underline{\beta}}) = \begin{bmatrix} 1.00 & -0.67 & -0.52 & -0.18 & 0.16 & 0.19 \\ & 1.00 & 0.91 & 0.21 & -0.18 & -0.22 \\ & & 1.00 & 0.21 & -0.19 & -0.22 \\ & & & 1.00 & -0.88 & -0.94 \\ & & & & 1.00 & 0.84 \\ & & & & & 1.00 \end{bmatrix} .$$

The adequacy of the competition parameters is also examined by dropping one parameter , two parameters etc., at a time by likelihood ratio criterion. The values of -2 log L and AIC under different constraints on the parameters are :

	Constraints						
	No Constraint	$\beta_{10}=\beta_{110}=\beta_{120}=\beta_{20}=\beta_{210}=\beta_{210}=0$	$\beta_{120}=0$	$\beta_{20}=0$	$\beta_{210}=0$	$\beta_{20}=0$ $\beta_{210}=0$	$\beta_{120}=0$ $\beta_{210}=0$
-2 log L	198.909	530.361	201.586	200.871	199.406	204.152	201.871
AIC	230.909	550.361	231.586	230.871	229.406	232.152	229.871

The value of -2 log L decreased considerably by fitting all the competition parameters. AIC is minimum for the model fitted with the constraint $\beta_{210} = 0$.

Hence this model is considered for further analysis though the increase in AIC is small when β_{120} and β_{210} both are taken as zeroes. The estimates of the parameters when $\beta_{210} = 0$ with their asymptotic standard errors are

Parameter	β_{10}	β_{110}	β_{120}	β_{20}	β_{220}	φ_{10}	φ_{20}	V_{10}	V_{20}
Estimate	-0.416	-0.192	-0.077	0.040	-0.491	3.149	6.825	0.219	0.432
Asymptotic standard error	0.100	0.102	0.024	0.016	0.036	0.352	0.330	0.053	0.096

The observed and expected yields, averaged over the replications, along with the residuals for both the crops in MC and IC, are given in Table 6.1.

The goodness of fit of this model is examined by plotting the residuals against the expected values (Figure 6.2). From this figure it is evident that the variability is increasing with the expected yields, which is expected from the model. Otherwise residuals do not show any pattern and thus the model appears to be satisfactory for these data. In some cases of mustard the data do not follow smooth pattern as expected, this is perhaps due to drought experienced by this crop (Kushwaha, 1983). Consequently, some of the residuals are large.

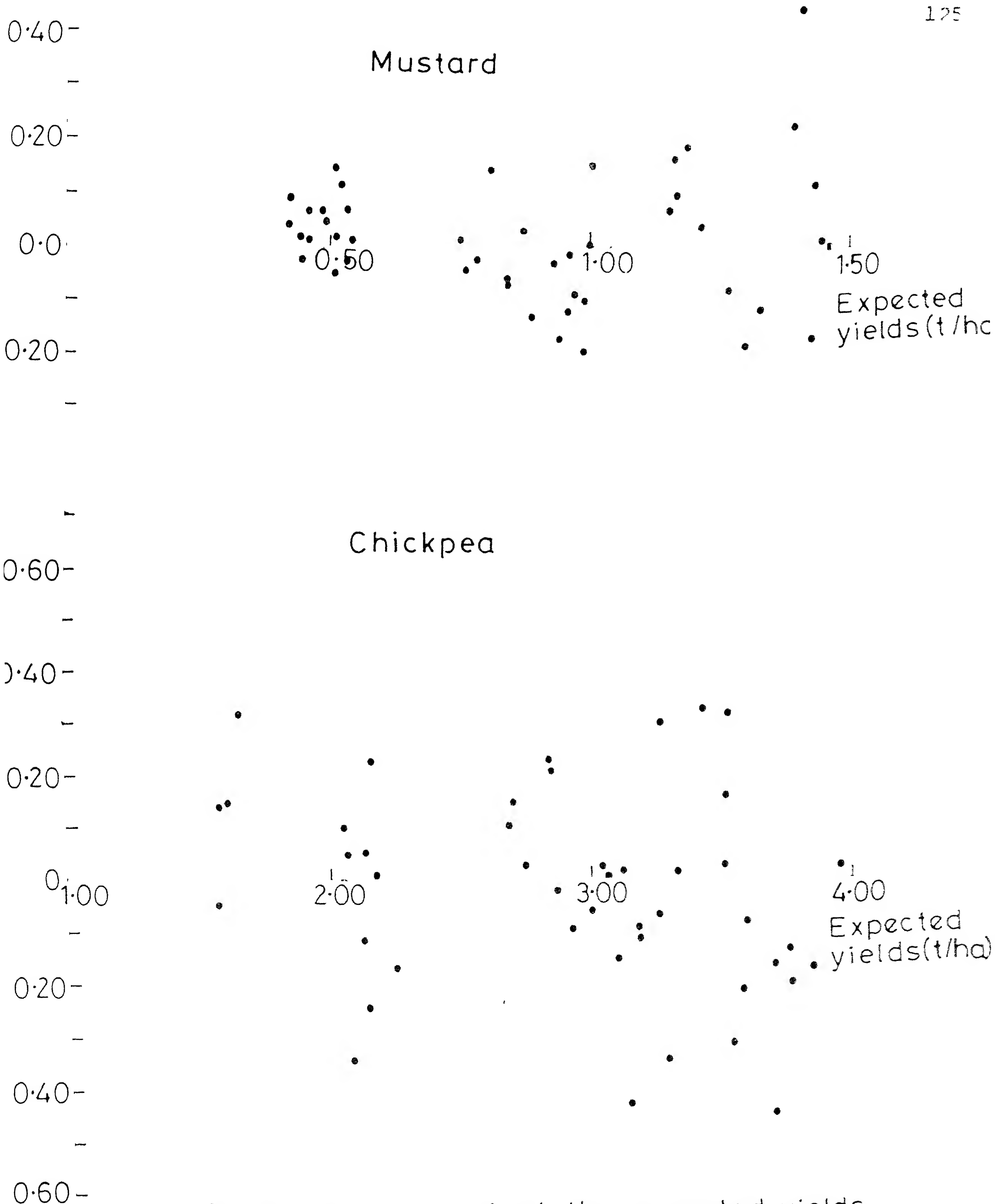


Fig. 6·2 Residuals against the expected yields for the fitted model

Table 6.1 Observed

Plant densities	Observed
1	1.614
2	1.233
3	1.233
4	1.274
1	0.639
2	0.514
3	0.526
4	0.464
1	0.958
2	0.784
3	0.802
4	0.771

and expected mean yields and residuals (t/ha).

Mustard		Chickpea		
Expected	Residuals	Observed	Expected	Residuals
	MC			
1.422	0.192	3.464	3.587	-0.123
1.419	-0.186	3.723	3.717	0.006
1.292	-0.059	3.717	3.574	0.143
1.197	0.077	3.466	3.321	0.145
	IC			
	1:4			
0.516	0.123	2.876	2.746	0.130
0.523	-0.009	2.971	3.184	-0.213
0.476	0.050	3.032	3.093	-0.061
0.432	0.032	2.917	2.909	0.008
	2:2			
0.976	-0.018	1.753	1.612	0.141
0.961	-0.177	2.341	2.103	0.238
0.860	-0.058	2.216	2.425	-0.209
0.772	-0.001	2.023	2.121	-0.098

6.3 DISCUSSION

Significance tests for the estimates based on their asymptotic variances and likelihood ratios give similar conclusions. However, Akaike's criterion appears to be a useful tool for selecting the parsimonious models although it does not indicate whether the better of the two models is significantly better. Problems in convergence for obtaining the ML estimates iteratively have been experienced in certain cases. This is because of high correlations among the estimates due to limited number of row arrangements and density levels used in the experiment. Hence one has to look for certain designs such that the correlations among the estimates is small. Detailed analysis of residuals has not been attempted because of the limited data. Looking at the plot of the residuals there (Figure 6.2) does not appear to be any pattern and the fit appears to be satisfactory. However, to establish the goodness of fit of the models developed here more experiments need to be analysed.

The estimates of the competition coefficients, broadly indicate the nature and degree of competition among the crops. The interspecific competition is considerably smaller than intraspecific competition among the plants of both the crops. The estimates of intraspecific competition coefficients are

$$\hat{\beta}_1 = (\hat{\beta}_{10} + \hat{\beta}_{110}) = -0.608, \quad \hat{\beta}_2 = (\hat{\beta}_{20} + \hat{\beta}_{220}) = -0.451,$$

for mustard and chickpea, respectively. Competition among mustard plants is more than in chickpea since $\hat{\beta}_1 = 1.35\hat{\beta}_2$. The estimates of interspecific competition coefficients are

$$\hat{\beta}'_{120} = \varphi_{20}\,\varphi^{-1}_{10}\,\hat{\beta}_{120} = -0.166,\ \hat{\beta}'_{210} = \hat{\varphi}_{10}\,\hat{\varphi}^{-1}_{20}\,\hat{\beta}'_{210} = 0,$$

for mustard and chickpea, respectively. There exists a significant interspecific competition for mustard whereas it is negligible for chickpea.

Optimum combination of row arrangement and plant densities can be obtained by adopting the methods discussed in the fifth chapter. In the case of MC optimum density levels for obtaining the maximum yields are

$$d^*_{10} = 1.394 \text{ and } d^*_{20} = 1.700,$$

for mustard and chickpea, respectively. The maximum expected yields corresponding to these densities are 1.47 and 3.87 t/ha. Here $\hat{\beta}_{20}$ is positive which is rather unexpected. This is perhaps due to limited amount of information available on the row arrangements. However, this is nearly nonsignificant. In extrapolation of the yield corresponding to row arrangements, particularly when $r_2 = 1$, β_{20} plays a dominant role. When it is positive then the expected yield for chickpea, when $r_2 = 1$ behaves in a very unexpected fashion, and consequently it has not been considered for optimization purposes.

The optimum densities in IC which maximize LER_e (5.1.1a) for various row arrangements, except when $r_2 = 1$, are the following :

Table 6.2 Optimum densities d_1^* (mustard) and d_2^*(chickpea)

Sl.No.	r_1	r_2	d_1^*	d_2^*	LER_e^*
1	1	2	3.601	2.043	1.326
2	1	3	2.283	1.595	1.235
3	1	4	2.276	1.660	1.190
4	2	2	2.003	1.609	1.197
5	2	3	1.987	1.666	1.153
6	2	4	1.980	1.718	1.128
7	3	2	1.867	1.636	1.143
8	3	3	1.854	1.687	1.115
9	3	4	1.848	1.736	1.099
10	4	2	1.788	1.648	1.112
11	4	3	1.776	1.697	1.092
12	4	4	1.772	1.744	1.081

These results give some indication for the future direction of experimentation. As we increase r_1 and r_2 LER_e decreases. Among the above combinations 1:2 row arrangement appears to be better. The optimum density levels are within

the range of the observed densities, and they depend upon the row arrangement. They are relatively larger when the crops are more intimate, consequently the values of LER_e are also larger. This indicates that more investigations including 1 : 1 arrangement are required.

Plot yields (t/ha) of each crop over the row arrangements $r_1 : r_2$ and plant densities ρ_1, ρ_2 (plants/m^2) (Kushwaha, 1983).

Row arrangement $r_1 : r_2$	Plants/m^2 ρ_1	Plants/m^2 ρ_2	Rep.1 Mustard	Rep.1 Chickpea	Rep.2 Mustard	Rep.2 Chickpea	Rep.3 Mustard	Rep.3 Chickpea	Rep.4 Mustard	Rep.4 Chickpea
MC	7.5	-	1.618	-	1.854	-	1.461	-	1.552	-
	15.0	-	1.098	-	1.113	-	1.460	-	1.261	-
	22.5	-	1.423	-	1.194	-	1.209	-	1.108	-
	30.0	-	1.221	-	1.257	-	1.245	-	1.373	-
	-	7.5	-	3.692	-	3.258	-	3.383	-	3.522
	-	15.0	-	3.583	-	4.000	-	3.675	-	3.633
	-	22.5	-	3.233	-	3.550	-	3.523	-	3.852
	-	30.0	-	3.571	-	3.762	-	3.342	-	3.187
IC 1:4	7.5	7.5	0.652	2.858	0.659	3.062	0.604	2.783	0.641	2.802
	15.0	15.0	0.461	2.708	0.447	2.944	0.550	3.086	0.496	3.146
	22.5	22.5	0.478	3.050	0.532	3.079	0.541	2.950	0.551	3.049
	30.0	30.0	0.463	2.843	0.507	2.940	0.424	2.821	0.462	3.052
2:2	7.5	7.5	0.841	1.727	0.954	1.989	1.152	1.758	0.884	1.537
	15.0	15.0	0.774	2.176	0.696	2.398	0.787	2.768	0.879	2.021
	22.5	22.5	0.779	2.196	0.774	2.083	0.752	2.696	0.901	1.887
	30.0	30.0	0.722	2.142	0.758	2.194	0.935	2.017	0.752	1.739

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Notation

β_{kk} autoregressive coefficient represents the effect of crop k row on the same crop

β_{km} $k \neq m = 1,2$ autoregressive coefficient represents the competition effect of crop m on crop k

β_{ko} autoregressive coefficient represents the competition effect between two neighbouring plants of crop k.

β_{kko} represents the competition between the two neighbouring rows of crop k at plant density of crop k $d_k = 1$

β_{kmo} $k \neq m = 1,2$ represents the competition of crop m on crop k when the plant density of crop m, $d_m = 1$.

Corrections

1.	Page 9 and 10	:	the section numbers should be 1.2.2.3 and 1.2.2.4 instead of 2.2.2.3 and 2.2.2.4.
2.	Page 13, line just above (1.2.13)	:	σ_i^2 not σ^2
3.	Page 21 line 3	:	When 'not' where
	line 7	:	y_{m,r_1+r_2+1} not $y_{m,r+r_2+1}$
4.	Page 23 line 6	:	The correct specification of Y_k and ε_k are $Y_1 = \sum_{i=1}^{n} y_{1i},\ Y_2 = \sum_{i=r_1+1}^{r_1+r_2} y_{2i}$; $\varepsilon_1 = \sum_{i=1}^{n} u_{1i},\ \varepsilon_2 = \sum_{i=r_1+1}^{r_1+r_2} u_{2i}$
5.	Page 24, (2.2.9)	:	$HA'\underline{1}$ not $H'A\underline{1}$
6.	Page 25, (2.2.11)	:	u_{ki} not u_{1i}
	line 9	:	u_{ki} not ε_{ki}
	line 15	:	$Y_1 = \sum_{i=1}^{N_1} Y_{1i}$: $Y_2 = \sum_{i=N_1+1}^{N_1+N_2} Y_{2i}$; $\varepsilon_1 = \sum_{i=1}^{N_1} u_{1i},\ \varepsilon_2 = \sum_{i=N_1+1}^{N_1+N_2} u_{2i}$ where $N_k = br_k$, $k = 1,2$.
7.	Page 26	:	In the matrix B, (1,2) element is β_{11} not β_{111}
	line 14	:	$\underline{y}$, $\underline{n}$ and $\underline{u}$ not y , n and u .

Page 54 line 9	:	Satisfactorily not satisfactory
Page 62 line 13	:	and not an
Page 63(4.2.7b)	:	n_2 not n_1
line 9	:	$(1-2\beta_k')$ not $(1-\beta_k)$ at all the three places
Page 71 line 3	:	$\underline{\varphi}_o = (\varphi_{10}, \varphi_{20})'$
Page 73 (4.3.11)	:	$\sum_k \sum_{m_k} \log b_{kom_k}$ not $\sum_k \sum_j \log b_{koj}$
Page 97	:	(4.2.25) not (4.2.2.5)
Page 123	:	In 3rd column of the table one of the β_{210} is β_{220}
Page 126	:	In column 6 last but one figure it is 2.177 not 2.425
Page 128 line 4	:	$\beta_{210}' = \varphi_{10} \varphi_{20}^{-1} \beta_{210}$

Page 27 line 4	:	$\underline{\varepsilon}$ not ε
line 5	:	$p_k = 1/2$ not $\mu_k/2$
Page 28	:	In matrix H in (2,2) element it is (r_2-1) not (r_1-1)
Page 43 (3.2.7)	:	β_{11} not β_1
Page 46 line 9	:	$\underline{\mu}$ not μ
Page 47 line 5	:	In the expression W one right parenthesis is extra and in M it is $\mu_k \partial \mu_m$ not $\partial^2 \mu_k \partial u_m$
Page 49 line 2	:	$E(B(p,q))$ not $E(B(p,q)$
Page 50 (3.4.3a)	:	$(\widehat{\mu_1+\alpha_{1j}})$ not $(\widehat{\mu_i+\alpha_{ij}})$
Page 51 line 7	:	3:3 is missing

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